DYNAMIC STABILITY ANALYSIS OF A SANDWICH BEAM WITH FUNCTIONALLY GRADED MATERIAL CONSTRAINING LAYER

A thesis submitted in the partial fulfilment

of the requirements for the degree of

Master of Technology

in

Mechanical Engineering

(Specialization: Machine Design and Analysis)

Submitted by

MALLIKARJANA REDDY OBBINENI

(Roll No: 211ME1159)

DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA-769008, INDIA.

MAY-2013

DYNAMIC STABILITY ANALYSIS OF A SANDWICH BEAM WITH FUNCTIONALLY GRADED MATERIAL CONSTRAINING LAYER

A thesis submitted in the partial fulfilment

of the requirements for the degree of

Master of Technology

in

Mechanical Engineering

(Specialization: Machine Design and Analysis)

Submitted by

MALLIKARJANA REDDY OBBINENI

(Roll No: 211ME1159)

Under the esteemed guidance of

Prof. S. C. MOHANTY

DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA-769008, INDIA

MAY-2013

NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA

CERTIFICATE

This is to certify that the thesis entitled, "**DYNAMIC STABILITY ANALYSIS OF A SANDWICH BEAM WITH FUNCTIONALLY GRADED MATERIAL CONSTRAINING LAYER**" submitted by Mr.**mallikarjanareddy obbineni (211me1159)** in partial fulfilment of the requirements for the award of **Master Of Technology** degree in **Mechanical Engineering** with specialization in **Machine Design and Analysis** at the National Institute of Technology, Rourkela (India) is an authentic Work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date:

Dr. S. C. Mohanty
Assoc. Professor
Department of Mechanical Engineering
National Institute of Technology
Rourkela-769008.

ACKNOWLEDGEMENT

It is my pleasure to have this opportunity to express my heartfelt and deep regards to those who have been helped me in the way of part of my life journey as student.

In particular, I would like to express my gratitude and regards to my thesis guide **Dr.S.C.Mohanty** for his valuable guidance, encouragement, and kind cooperation in the successful completion of work.

I also would like to express my sincere thanks to **Prof. K.P.Maity** Head of the department, Mechanical Engineering for providing valuable departmental facilities.

I also thankful to research scholars Raghavendhra, Ramu and Pavankishore of Mechanical engineering department for their support.

Finally, I would like to thank my fellow post graduate students and the people who have been involved directly or indirectly in my endeavour.

MALLIKARJANA REDDY OBBINENI

Roll No: 211me1159

Dept.Of Mechanical Engg.

Contents

ABSTRACT

The present work aims to study the dynamic stability of a three layer sandwich beam with viscoelastic core and functionally graded material constraining layer. Sandwich beam is modelled as a line element having two nodes of each having four degrees of freedom. Finite element method is used to model the beam. Variation of properties of functionally graded material is taken according to a simple power law. The Hsu's procedure proposed by Saito and Otomi has been used to determine boundaries of stable and unstable regions of the sandwich beam. The effect of various system parameters such as core thickness ratio, power law index, core loss factor etc., on the dynamic stability of the sandwich is studied theoretically.

For the fixed- free sandwich beam buckling load decreases with increase in power law index. The frequency parameter first decreases and then increases for increase in power law index and core thickness ratio parameter. Fundamental loss factor of sandwich beam increases with increase in the core loss factor. Increase in core thickness ratio, first decreases and then increases the fundamental loss factor. Increase in power law index values, shows more probability of instability of beam. Increase in thickness ratio also enhances the instability chances of sandwich beam.

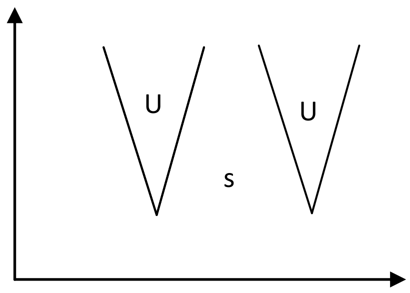
Key words: Dynamic stability, Sandwich beam, Constraining layer, Functionally Graded material.

List of Figures

Nomenclature

A_k	:	Cross –sectional area of the k^{th} elastic layer.
A_v	:	Cross –sectional area of the viscoelastic layer.
E_k	:	Young's modulus of the k^{th} elastic layer.
g	:	Shear parameter.
G_v	:	Complex shear modulus of the viscoelastic layer.
h	:	Thickness of FGM layer
t_2	:	Thickness of the viscoelastic layer
I_k	:	Moment of Inertia of the k^{th} elastic layer.
$\left[K^{(e)}\right]$	:	stiffness matrix of the beam element
$\left[K_p{}^{(e)}\right]$	:	stability matrix of the beam element
kd_1	:	distance between neutral plane and mid plane
L	:	Length of the beam.
Le	:	Length of the beam element.
[M]	:	Global mass matrix.
n	:	Power law index
$[N_k]$	:	Shape function matrix of the k^{th} elastic layer for axial
$[N]^T$	:	Transpose of shape function matrix.
$[N_v]$	:	Shape function matrix for viscoelastic layer.
$[N_w]$	:	Shape function matrix for transverse displacement.
P_0	:	Static component of the load.
P_l	:	Time dependent component of the load.
P(t)	:	Axial periodic force.
P_{cr}	:	Critical buckling load of the equivalent Euler beam.
$T_k^{(e)}$	:	Kinetic energy of the beam element.
$T_v^{(e)}$	:	Kinetic energy of the viscoelastic layer.
u_k	:	Axial displacement of the k^{th} elastic layer.
$U_k^{(e)}$	:	Potential energy of the constraining layer.
$U_v^{(e)}$	:	Potential energy of the viscoelastic layer.
V_c	:	volume fraction of the ceramic material.
V_m	:	volume fraction of the metal.
w	:	Transverse displacement.

$W_p^{(e)}$	:	Work done on the beam element.
x	:	Axial co-ordinate.
z	:	position from the reference plane
α	:	Static load factor.
β	:	Dynamic load factor.
ρ	:	Mass density of the elastic layer.
ρ_v	:	Mass density of the viscoelastic layer.
Ω	:	Distributing frequency.
η_c	:	Core loss factor.
Φ	:	Rotational angle.
$\xi = x/L_e$		
γ_v	:	shear strain of viscoelastic layer.
$\{\Delta^{(e)}\}$	:	Nodal displacement matrix of the beam element.
$[\Phi]$	:	Normalised modal matrix.
$\{\Gamma\}$	:	New set of generalized co-ordinates.
'	:	$\frac{\partial}{\partial x}$
.	:	$\frac{\partial}{\partial t}$
..	:	$\frac{\partial^2}{\partial t^2}$
S	:	Stable region.
U	:	Unstable region.



1.1 INTRODUCTION

Damping is very important in structures and systems subjected to dynamic loading. Passive damping treatment is the one of the ways to control the noise and vibration in structures. The airborne and structure borne noise and vibration are most frequent in many systems. The traditional passive control methods that include use of absorbers, barriers, mufflers, silencers, etc. are for airborne noise. For systems with constant excitation frequency, modification of system's stiffness or mass reduces the unwanted vibrations as these parameters alter the resonance frequencies. But in most cases, the isolation or dissipation of vibrations by using isolators or damping materials is needed. Advances in material technology and more sophisticated analytical and modelling techniques for the dynamic behaviour of materials and structures facilitates the engineers and led to many applications. Viscoelastic materials (damping material) exhibits both viscous fluid and elastic solid material characteristics. There are mainly two methods of treatment of viscoelastic material viz., constrained layer and unconstrained layer or free layer treatment. Depending on the functional requirements Sandwich structure utilizes the constrained layer treatment method in obtaining efficient properties of all the layers. In this case the viscoelastic material is sandwiched between the surface of structure and thin facings of metallic material.

The traditional Sandwich construction includes a relative thick core of low density material, sandwiched between the top and bottom face sheets (face layers) of relatively thin in size.

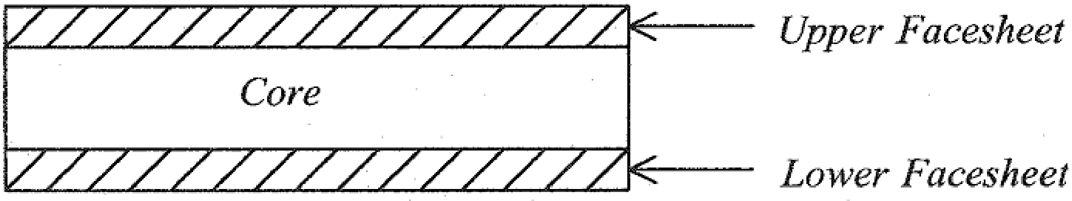


Fig: Sandwich model

1.2 MATERIALS & MATERIAL PROPERTIES OF SANDWICH STRUCTURE

Depending on the performing function in which sandwich beam is going to be used, the choice of materials is vast. The introduction and advancement of composite materials and functionally graded materials grease the wheels of researchers in choice of selection of materials for different applications. The choice of material in sandwich structure relies upon property needs of employment (high strength, high temperature resistivity, surface finish etc.). The number of available cores has increased greatly in recent times due to the introduction of more competitive cellular plastics. Combination options of the face sheet materials with different core materials enable new ideas to be integrated in a wide range of applications.

It is the obligation of the designer to have reliable information about the strength and stiffness of the materials used in the design for efficient analysis and design of sandwich structures. The best practise is to devote to tests for obtaining adequate material properties. The ample number of material choices may appear as an additional complexity but is really one the main features of using sandwich structures. The materials best suited for a particular application may be utilized and some drawbacks can be overcome by geometrical sizing. The elementary objective of the designer is to achieve an efficient design that will utilize each material component to perform the function with good efficiency.

1.2.1 CORE MATERIAL:

The core's function is to support the thin skins so they do not buckle (deform) inwardly or outwardly, and to keep them in relative position to each other. Key requirements for the core are normally the shear modulus and strength and compressive modulus. The basic objective of the designer in choice of core material is that it will not fail under the applied load. There should be no deformation of core, in thickness wise. Thus requiring a high modulus of elasticity perpendicular to the faces. The core is exposed to shear so that global deformations and core shear stresses are developed by the shear strains in the core. Weight of the core material such that least load is added to the total weight of the structure.

The core material and thickness of the core are two main parameters that decide the most of the properties (thermal, acoustical, damping) of the sandwich structure. The following are the typical features of the core material;

- Low density
- Dampening of vibration and noise
- Shear modulus and shear strength
- Stiffness perpendicular to the faces
- Thermal insulation

A variety of materials find application as the core in sandwich structures:

Polymeric foam cores: Foams are one of the most common forms of core material and may be either closed cell or open cell. A wide variety of polymeric foams are available with unique properties, different types are as follows

Polyethylene terephthalate (PET): these foams are easily machined and thermoformed, the main characteristics are good compression strengths and moduli, fatigue resistance, creep characteristics and thermal stability to 150ºC

Polymethacrylimide (PMI): foams combine the highest overall strength and stiffness for foam cores of a given density with high dimensional stability, high fatigue life and can be used at elevated temperatures. The drawback is the high cost which has limited their use to high performance composite components: helicopter rotor blades, ailerons and stringer profiles in pressure bulkheads.

Polyvinylchloride (PVC): are commonly used as core materials for high performance sandwich structures. They are usually a hybrid of PVC and polyurethane rather than pure PVC. They offer a balance of static and dynamic properties with good resistance to water absorption.

Syntactic cores: Syntactic foam is a lightweight composite consisting of hollow spheres in a resin matrix. These hollow spheres are normally polymeric or glass

Wood cores: End Grain Balsa (EGB) is an ultra-light wood product with one of the highest strength to weight ratios for a core material.

Honeycomb and corrugated cores:

Many different materials may be used for honeycomb cores with the most common being aluminium, paper or polymers. Honeycomb cores are made by selectively bonding layers of scored material and then expanding the stack to produce a regular cellular structure. Alternative routes for their manufacture include corrugation followed by bonding and the extrusion of thermoplastics.

1.2.2 FACE MATERIAL

The top and bottom layers of conventional sandwich structure are called as face layers or face sheets (as layers are in sheet form). Face materials can be obtained from any structural material that can be available in the form of thin sheets. The faces carry the tensile and compressive stresses in the sandwich. The local flexural rigidity is often so small it can be ignored. It is also acceptable and common to choose fibre glass-reinforced plastics as face materials.

- High impact resistance
- High tensile and compressive strength
- Wear resistance
- Resistance to different conditions (chemical, heat, etc.)
- High stiffness giving high flexural rigidity
- Good surface finish

Various types of materials used as face materials are as follows:

Metals and alloys: Metals and their alloys possess all most all required properties of face materials. Conventional materials and their alloys such as steel, stainless steel and aluminium are often used as face material. As the thickness of the face layers are relatively thin so one limitation for use of metals is that those can form into sheets are used.

Composites: Most composites offer properties similar to or even higher than those of metals, they have been substantially used in construction of sandwich structures. Particularly fibre reinforced composites are suitable for sandwich structures even though the stiffness is often lower in magnitude. Thus with a light core, the composites produce high rigidity.

Functionally graded materials, we may say special type composites, are especially applicable for some conditions. Ceramic-metal FGMs are best example for high temperature environment conditions. Exclusive properties of FGMs made them more suitable for specific applications as face layers of sandwich structures.

Even wood also can used as face material in sandwich structures.

1.2.3 Design Considerations

A sandwich structure is designed to make sure that it is capable of taking structural loads throughout its design life. In addition, it should maintain its structural integrity in the in-service environments. The structure should satisfy the following criteria:

- The face sheets should have sufficient stiffness to withstand the tensile, compressive, and shear stresses produced by applied loads.
- The core should have sufficient stiffness to withstand the shear stresses produced by applied loads.
- The core should have sufficient shear modulus to prevent overall buckling of the sandwich structure under loads.
- Stiffness of the core and compressive strength of the face sheets should be sufficient to prevent the wrinkling of the face sheets under applied loads.
- The core cells should be small enough to prevent inter-cell buckling of the face sheets under design loads.
- The core shall have sufficient compressive strength to prevent crushing due to applied loads acting normal to the face sheets or by compressive stresses produced by flexure.
- The sandwich structure should have sufficient flexural and shear rigidities to prevent excessive deflections under applied loads.
- Sandwich materials (face sheet, core and adhesive) should maintain the structural integrity during in-service environments.

1.2.4 APPLICATION AREAS OF SAND STRUCTURES:

In damped structures for effective vibration damping

Aerospace field

Building Construction

Naval ships

Rail Industry

Automotive Industry

1.3 PRESENT CONSIDERATION:

In the present analysis considered sandwich structure consists of

Upper layer (top face): Functionally graded material

Middle layer (core): Viscoelastic material

Lower layer (bottom face): Metallic

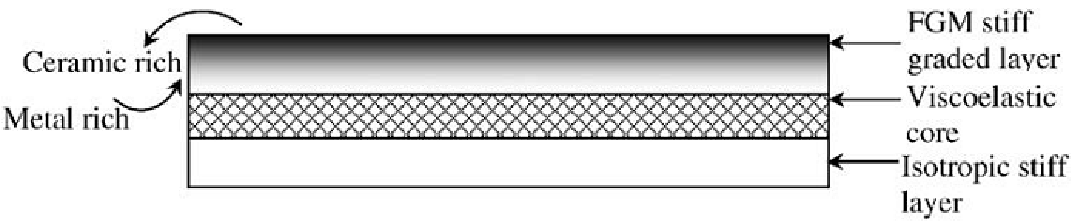


Fig-2: sandwich beam with FGM constraining layer

1.4 Functionally graded material (FGM):

In this fast paced world, everything needs to be customized which intern requires improvement of materials for certain applications to become more efficient. After all benefits of composites some limitations questions the importance of more functional oriented materials existence, nothing but Functionally Graded Materials.

Functionally Graded materials are belongs to family of composites. The FGM concept was first proposed by group of scientists in Japan, in 1984[1] as a solution of a thermal problem.

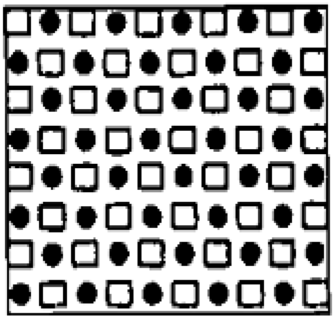


Fig: composites

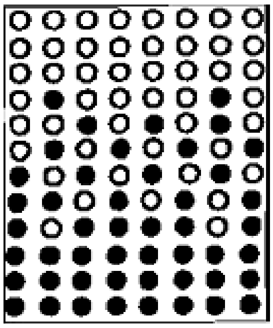


Fig: FGM

Metal ● Ceramic ○ Fiber □

Fig.3: configurations of composites and FGM

The FGMs properties vary from one surface to another smoothly or gradually based on the spatial position throughout the thickness. Most frequently FGMs made up of ceramic and metal material components. Generally one surface maintains the homogenous material property of ceramic or metal. The other opposite face is also same in properties wise with other material based on the application.

The main role of the metal constituent in the FGM is to provide the structural support, while the other constituent ceramic is to provide heat shielding or thermal barrier when subjected to high temperature environments. The material property variation makes it possible to accommodate the function to appropriate the needs of different applications. FGMs offer great agreement in applications, where the operating conditions are more difficult. The concentrations of material components are different for various functions. High amount of ceramic material or pure ceramic is needed to arrange at the upper face at high temperature conditions, corrosive environments, in the same way high amounts of metal are provided at locations where mechanical properties such as high strength, toughness etc to be needed.

The FGM concept is not entirely new, keen observation of some natural things reveals the truth, that nature has been using the FGM concept. Some examples are bamboo in which spatial variation of voids and pores concentration makes them rich in bending rigidity with minimum mass. Human skin and Bones are some more examples of natural FGMs.

1.4.1 APPLICATIONS

The following are the various application areas of functionally graded materials

1. Aerospace:

Functionally graded materials can be used under high temperature conditions with one of its constituents as low thermal conductivity material. They can resist high thermal gradients, this makes functionally graded materials appropriate for structures in aerospace such as space plane body, rocket engine components e.t.c

2. Medicine

Living tissues like bones and teeth are described as natural functionally graded materials. A suitable material is required as substitute to these living tissues, if the original one not serving purpose. The ideal one for this application is functionally graded material. FGM has find wide range of application in dental and orthopaedic applications for teeth and bone replacement.

3. Defence

In defence application, such as armour plates and bullet-proof vests, penetration resistant materials are needed. One of the most important characteristics of functionally graded material is the ability to inhibit crack propagation, this makes functionally graded materials suitable for defence applications.

4. Energy

FGM are used in energy conversion devices. They also provide thermal barrier and are used as protective coating on turbine blades in gas turbine engine.

5. Optoelectronics

FGM also finds its application in optoelectronics as graded refractive index materials and in audio-video discs magnetic storage media.

Other areas of application are: cutting tool insert coating, automobile engine components, nuclear reactor components, turbine blade, heat exchanger, Tribology, sensors, fire retardant doors, etc. The list is endless and more application is springing up as the processing technology, cost of production and properties of FMG improve.

1.4.2 Limitations

Various processing methods are available today for manufacturing of functionally graded materials almost any material combination. Regardless of all achievements there will be new challenges in future when applications for FGMs will enlarge.

1. Conversion of manufacturing processes to mass production.
2. Cost-effectiveness of production processes
3. Quality control.
4. Repeatability of production processes and reliability of the produced FGMs.

LITERATURE REVIEW

The first measurable analysis on the effectiveness of damping of a constrained viscoelastic layer was done by Kerwin [19] and also he acquired an expression for the loss factor estimation. For the uniform linear composites Unger [2] derived the general expressions for calculating the loss factor in terms of the properties of constituting materials. A theory developed by Di Taranto [3] estimates the natural frequencies and loss factors for a finite length sandwich beam. Jones et al. [13] analyzed the damping capacity of viscoeastic core both theoretically and experimentally. Multi layer sandwich beams with simply supported boundary conditions analyzed by Asani and Nakra [4] and they estimated displacement response effectiveness and loss factors for the same. The natural frequencies and logarithmic decrement for the fundamental mode of vibration for a simply supported sandwich beam is estimated by Chatterjee and Baungarten [5] . They also verified the theoretical results with experimental ones with good agreement. The vibration characteristics of asymmetric dual core sandwich beam studied by Nakra and Grootenhuis in both theoretically and experimentally without considering the rotary and longitudinal inertia terms. Rao [6] studied previous problem with including both effects. Rao through his investigation on the influence of pretwist on resonant frequency and loss factor for a symmetric pretwisted s.s.sandwich beam, concluded that loss factor reduces with pretwisting, the effect of pretwist is more sensitive in case of very soft thick cored beams. Asani and Nakra [7] studied the variation of system loss factor with number of layers and thickness ratio in a s.s.multilayer beam. Rubayi and Charoenee [8] found the natural frequencies of sandwich beam with cantilever arrangement by theoretical and experimental investigations. Rao and stuhler [9] analyzed tapered sandwich beam damping effectiveness with simply supported and cantilever end conditions. Rao [10] studied the free vibration of a short sandwich beam including all effects such as inertia, extension and shear of all layers. And he estimated a maximum error of 45% in loss factor and frequencies calculation if the parameters neglected. For sandwich beam Rao [11] gave equitation and graphs for frequencies and loss factor estimation with different boundary conditions. The equation of motion for a sandwich beam with harmonic excitation derived by Vaswani et ai. [12]. Johnson and his co-workers [14-15] solved beams and plates with constrained layer viscoelastic layer frequencies and loss factors using finite element method. Lall et al. [16] analyzed three different methods of partially covered sandwich beams concluded that Marcus [17] method estimates modal loss factor only, but Rayleigh-ritz and classical search methods find both loss factor and resonant frequencies. Dewa et al. [18] examined partially covered sandwich beams damping effectiveness and also noticed that in damping capacity partially covered beams are superior than fully covered beams. He also justified the theoretical findings with experiments. Imaino and Harrison[21] Investigated Damping of first and second bending resonance of sandwich beam with constrained damping layer by implementing modal strain energy method and finite element technique. To execute a parametric study of the coupled flexural and longitudinal vibration of a curved sandwich beam Rao[22] developed a analytical model. And also they investigated effects of various parameters on system loss factors and resonant frequencies namely curvature, core thickness and adhesive shear modulus etc. In another work by same authors on multi span beam they studied the vibration behaviour with arbitrary end conditions. And also they investigated behaviour of loss factors and resonant frequencies with various parameters like location of intermediate supports and adhesive thickness. The resonant frequencies and loss factors are solved by Bhimaraddi [24] for constrained layer damping beam with simply supported end conditions using a model that includes continuity of displacements and the transverse shear stresses across the interfaces of the layers. An analytical model is developed by Sakiyama et al.[25] for the investigation of free vibration analysis of a three layer continuous sandwich beam and also variation of shear parameter and core thickness and their consequences on the resonant frequencies and loss factors. Rayleigh –Ritz method is used by Fasana and Marchesiello [26] for calculation of mode shapes, frequencies and loss factors for sandwich beams. Banerjee [27] studied free vibrations and calculated natural frequencies and mode shapes of a three layer sandwich using dynamic stiffness matrix method.

2.1 Static and dynamic analysis of sandwich beams:

The forced vibration analysis of a three layered sandwich beam with viscoelastic core done by Mead and Markus [28] with method used same as Di Taranto [29] for arbitrary end conditions. Asnani and Nakra [30] did same forced vibration analysis for the sandwich beam with viscoelastic core for fixed-free and fixed –fixed end conditions. And also they used Ritz method to get the forced vibration response and validated with experimental results. Rao [31] investigated the forced vibration of a damped sandwich beam considering moving forces. And also he found variation of dynamic magnification of the central deflection of beam is inversely proposal to shear difference of the core material. Kapur [32] analyzed the dynamic response of two and three layered viscoelastically damped beams under half-sine shock excitation. In his analysis he included both rotary and longitudinal inertia. Static deflection and stresses in sandwich beams were determined by Sharma and Rao [33]. With concentrated and distributed loads under various conditions. Frosting and Baruch [34] investigated stresses in sandwich beams with flexible core under concentrated and distributed loads. They found that in some cases of sudden failure of the beam is due to significant transverse normal stresses at the interface between the skin and core. Sun et al. [36] worked out finite element model to study the effect of viscoelastic layer in damping and vibration control of unidirectional composites. Their theoretical findings are well in agreement with the experimental results. Non-linear analysis of sandwich beam was done by Salet and Hamelink [37] for simply supported arrangement. And also they developed a numerical model based on finite difference method. Ha [38] analyzed and suggested an exact analysis method for buckling and bending analysis of sandwich beam systems. Qian and Demao [39] execute modal analysis as well as response calculation using finite element method.

2.2 Stability study of sandwich beams and column

Bauld [40] analyzed the sandwich columns subjected to pulsating axial loads with simply supported arrangement. Chonan [41] studied the stability of two layer sandwich cantilever beam with imperfect bonding. They found out that critical loads are functions of shear and normal stiffness of the bond for divergence and flutter type instabilities. For the same instabilities in another work author[42] found out that critical divergence and flutter loads depends on interface bond stiffness for the case of symmetric sandwich beams with elastic bonding. Kar and Hauger [43] found out the critical divergence and flutter loads in the investigation of dynamic stability of sandwich beam subjected to a direction controlled non conservative force. The dynamic stability of sandwich beams is investigated by Ray and Kar [44] for different boundary conditions. In another work by same authors [45-47] on partially covered dual cored sandwich beams and symmetric sandwich beams with higher order effects, they investigated the parameter stability. They derived governing equation of motion using Hamilton's principle; he used Galerkin's method for converting the equations of motion to a set of coupled hill's equation in the time domain. Lin and chen [48] studied the dynamic stability of rotating sandwich beam with constrained damping layer with parameters rotating speed, setting angle and hub radius and their effects.

2.3 Functionally graded materials:

Bhangale and Ganeshan [1] have investigated the static and dynamic behaviour of functionally graded material sandwich beam in thermal environment having constrained viscoelastic layer using finite element method. They found that materials having lower thermal coefficient of expansion possess high thermal buckling temperature. With increase in the value of power law index, the critical buckling temperature for an FGM sandwich beam increases. Aydogdu and Taskin [51] worked on free vibration analysis of functionally graded beams with simply supported edges. Kapuria et al, [20] investigated both the static and dynamic behaviour of FGM beams made of like Al/Sic and Ni/Al_2O_3 for different end conditions using zigzag theory. Simsek [52-53][23] used different higher order theories to study the dynamic analysis of FGM beams. Akhtar and Kadoli [54] presented the static behaviour of various FGM beams. Alshorbagy et al. [35] used principle of virtual work to study the dynamic characteristics of a functionally graded Euler-Bernoulli beam. Aminbaghai et al. [55] investigated the effect of large axial force on the free vibration of multilayer FGM beams under longitudinal variable elastic foundation. Chakraborty et al. [55] have studied the thermo elastic behaviour of functionally graded beam structures with exponential and power law variation of material properties along thickness and also developed a beam finite element to study the same.

2.4 BLUEPRINT OF THE PRESENT WORK:

The current work concerns study of the dynamic stability of the sandwich beam with functionally graded material constraining layer. The principal aim of the current work is to study the effect of various system parameters such as core thickness parameter, core loss factor, power law index etc., on the dynamic stability of the FGM constraining layer sandwich beam. Finite element method is used to model the sandwich beam. Modified Hsu's method proposed by Saito and Otomi has been used to establish the boundaries of instability regions of the sandwich beam. Dynamic stability of sandwich beam with fixed-free end condition subjected to axial periodic force has been reported. The effects of various parameters on the dynamic stability have been plotted.

3.1 BRIEF HISTORY OF DEVELOPMENT [56]

The current development of the finite element method started in 1940s in the field of structural engineering with the work by Hrennikoff in 1941 and McHenry in 1943,who used a frame of line (one dimensional) elements (bars and beams) for the solution of stresses in continuous solids. Courant proposed a variational form for the setting up of stresses. Later he presented piecewise interpolation (or shape) functions over triangular sub regions making up the whole region as a method to obtain approximate numerical solutions. In 1947 Levy developed a new method namely force (or flexibility) method, and the same author in his another work suggested one more method the stiffness (or displacement) method. In 1954 Argyris and Kelsey developed matrix structural analysis methods using energy principles. This development illustrated the important role that energy principles would play in the finite element method. The two dimensional elements are first treated by Turneret al. in 1956. They developed stiffness matrices for truss, beam elements and two-dimensional triangular and rectangular elements in plane stress. The development of high speed digital computer in the early 1950s Turneret al. prompted further development of finite element stiffness equations expressed in matrix notation. Extension of the finite element method to three dimensional problems with the development tetrahedral stiffness matrix was done by Martin in 1961. Up to early 1960s most of the finite element works are limited to small displacements and strains and static loadings. Later on in 1960 large deflection and thermal analysis and non linearities are considered by various authors. Zienkiewicz et al. extended the finite element method to viscoelasticity problems in 1968. During the decades of the 1960s and 1970s, the finite element method was extended to applications in plate bending, shell bending, pressure vessels, and general three dimensional problems in elastic structural as well as to fluid flow and heat transfer. Further development of the method to applications large deflection and dynamic analysis also occurred in these periods. The finite element method is computationally intensive, owing to the required operations on very large matrices. During 1960s the finite element software code NASTRAN was developed in conjunction with the space exploration program of the United States. NASTRAN was the first major finite element software package. From the early 1950s to present, enormous advances have been made in the application of the finite element method to solve complicated engineering problems.

3.2 Fundamental concept of FEM

The main rule that involved in finite element method is "DEVIDE and ANALYZE". The greatest unique feature which separates finite element method from other methods is "it divides the given domain into a set of sub domains, called 'finite elements'". Any geometric shape that allows the computation of the solution or its approximation, or provides necessary relations among the values of the solution at selected points called 'nodes' of the sub domain, qualifies as finite element. Division of the domain into elements is called 'mesh'. Approximate solutions of these finite elements give rise to the solution of the given geometry which is also an approximate solution.

The approximate solution becomes exact when

1. Division of the given domain into infinite number of sub domains or elements
2. The expression for the primary variable must contain a complete set of polynomials (infinite terms).

3.3 General steps of the Finite Element Method

The following general steps discussed below are for structural analysis case.

1.Discretize and select the element types:

This step includes dividing the body into an equivalent system of finite elements with associated nodes and selecting the best suited element which resembles the actual physical behaviour of the given system to be analyzed. Engineer needs to focus in the matters of selecting the number elements, variation in size and type of elements. For getting best results it is advisable to choose as small elements as possible. One of the major task of the engineer is the selection of the appropriate element for a particular problem.

2. Select a Primary variable function:

This step involves selecting a primary variable (displacement) function within each element. The function is defined within the element using the nodal values of the element. Polynomial functions are generally used because they are easy to work within finite element formulation. In case of two dimensional elements, the primary variable function is function of the coordinates in its plane. The functions are expressed in terms of the nodal unknowns.

3. Define relations:

The relations among stresses, strains and displacements are essential for obtaining the equations for each finite element. In the case of one –dimensional deformation, say, in the x direction, we have strain ε_x related to displacement *u* by

$$\varepsilon_x = \frac{du}{dx}$$

for small strain cases. The definition of material behaviour is also important in obtaining acceptable results.

4. Derive the element stiffness Matrix and Equations:

The element stiffness matrix and equations are deriving by using the following methods.

<u>Direct Equilibrium Method</u>

According to this method, the stiffness matrix and element equations relating nodal forces to nodal displacements are obtained using force equilibrium conditions for a basic element, along with force or deformation relationships. This method can easily adaptable to line or one-dimensional elements.

Work or Energy Methods

It is much easier to apply a work or energy method to develop the stiffness and equations for two and three-dimensional elements. The principle of minimum potential energy, the principle of virtual work methods used for derivation of element equations.

The principle of virtual work is applicable for any material behaviour, whereas the principle of minimum potential energy is applicable only to elastic materials.

Methods of Weighted Residuals

The methods of weighted residuals are useful for developing the element eqations, particularly popular is Galerkin's method. These methods yield the same results as the energy methods wherever the energy methods are applicable. They are especially useful when a functional such as potential energy is not readily applicable.

5. Assembling the Element equations and introduce boundary conditions:

In this, the individual element nodal equilibrium equations generated in previous step are assembled into the global nodal equilibrium equations. One more direct method of superposition, whose basis is nodal force equilibrium, can be used to obtain the global equations.

The global equation can be written in matrix form as

$$\{F\} = [K]\{\delta\}$$

Where $\{F\}$the vector of global nodal forces, $[K]$ is the total stiffness matrix $\{\delta\}$ is the vector of generalised displacements.

At this stage, the global stiffness matrix $[K]$is a singular matrix because its determinant is equal to zero. To remove this we need to call upon certain boundary conditions in order to avoid to the movement of the structure as rigid body.

6. Solve for the primary unknowns:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{1n} \\ K_{21} & K_{22} & K_{2n} \\ K_{n1} & K_{n2} & K_{nn} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_n \end{Bmatrix}$$

These general equations can be solved for the primary unknowns by using an elimination method or an iterative method. The primary variables are different for various problems. In case of the structural problem the primary unknown is displacement.

7. Solve for secondary unknowns:

The secondary variables are different for various applications. For the structural stress-analysis problem, important secondary quantities are strain and stress (or shear force and moment) can be obtained by using the already established relations expressed in terms of the displacements determined in the previous step.

8. Interpret the results:

The final aim is to solve and analyze the results for use in the design or analysis process. For better design and to avoid the failure of the structure it is important to determine the locations in the structure where large deformations and stresses are occur. Postprocessor computer programs help the user to interpret the results by displaying them in graphical form.

3.4 Applications of the Finite Element Method

The finite element method is an effective tool to analyze both structural and non-structural problems. Even in some biomechanical engineering problems, includes analyses of human spine, hip joints, skull, hip joints, heart and eye e.t.c

Structural areas:

1. stress analysis, including truss and frame analysis, and stress concentration problems commonly associated with holes, fillets, or other changes in geometry in a body.

2. Buckling analysis

3. Vibration analysis

Non-structural problems

1. Heat transfer

2. Fluid flow

3. Distribution of electric or magnetic potential

3.5 Advantages of the Finite Element Method

Some of the advantages of the Finite element method that includes the ability to

1. Easy modelling of irregularly shaped bodies.

2. Handle general load conditions without difficulty.

3. Model bodies composed of different materials

4. Handle unlimited numbers and kinds of boundary conditions

5. Includes dynamic effects

6. Handle nonlinear behaviour with large deformations and nonlinear materials.

3.6 Limitations of the Finite Element Method

In spite of many advantages some drawbacks of finite element method are as follows

1. Stress values depend on the size of mesh fine to average.

2. In some cases the approximations used may not provide accurate results.

3. For vibration and stability problems the cost of analysis by FEA is prohibitive.

Software packages for FEM

Below list provides some of the commercially available software packages of FEM

ABACUS

ANSYS

COSMOS/M

LS-DYNA

NASTRAN

4.1 ENGINEERING MODEL OF FGM MATERIAL PROPERTIES

Most often FGMs are produced by using ceramic-metal material constituents. Ceramic material constituents take care of high temperature conditions by providing heat shield as its thermal conductivity is low. The other one i.e. metal material constituents arrests fracture originated by stresses due to high thermal gradient.

In the present study, in FGM layer the variation of material constituents of ceramic (Alumina) and metal (steel) are from top to bottom surfaces respectively through the thickness direction only according to simple Power law. Top surface of beam is of pure ceramic and bottom surface is of pure metal material.

The positions (Z) of top and bottom surfaces are denoted by (z = h/2) to (z = –h/2) respectively from the reference plane at the middle of the FGM layer where (z = 0). The volume fractions of the material constituents of ceramic and metals are denoted by V_c and V_m respectively. Sum of the volume fractions of FGM is equal to unity, mathematically

$$V_c(z) + V_m(z) = 1 \tag{4.1}$$

The effective material properties of FGM constituents are graded using simple power law. According to power law the volume fractions of the ceramic and metal are given by the following relations

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n \tag{4.2}$$

Where n = power law index

h = thickness of FGM layer

z = position from the reference plane

From eq.(1) we can get the metal volume fraction

$$V_m(z) = V_c(z) - 1 \tag{4.3}$$

Where n designate the material variation profile along the thickness of FGM layer. It is a non negative volume fraction index varies from $0 \leq n \leq \infty$

The resultant properties like Elastic modulus(E), density(ρ), Poisson's ratio(ν), thermal conductivity(k), Thermal coefficient of expansion(α) etc., at any particular chosen position can written by the following equations

$$E_{eff} = (E_m - E_c)\left(\frac{2z+h}{2h}\right)^n + Ec \tag{4.4}$$

$$\rho_{eff} = (\rho_m - \rho_c)\left(\frac{2z+h}{2h}\right)^n + \rho_c$$

$$\nu_{eff} = (\nu_m - \nu_c)\left(\frac{2z+h}{2h}\right)^n + \nu_c$$

$$\alpha_{eff} = (\alpha_m - \alpha_c)\left(\frac{2z+h}{2h}\right)^n + \alpha_c$$

$$k_{eff} = (k_m - k_c)\left(\frac{2z+h}{2h}\right)^n + k_c$$

4.2 Mathematical Modelling of the problem:

A three layered symmetric sandwich beam of length L, subjected to a pulsating axial force $P(t)=P_0+P_1\cos\Omega t$, is acting along its un deformed axis at one end. Where Ω is the distributing frequency. P_0 and P_1 are the amplitudes of static and time dependent component of the load respectively. Figure shows the sandwich beam.

The following assumptions have been considered in the development of finite element model:

1. The transverse displacement is w is same for all the three layers.
2. Linear theories of elasticity and viscoelasticity are used.
3. The rotary inertia and shear deformation in the constrained layers are negligible.
4. No slip occurs between the layers and there is perfect continuity at the interfaces.
5. Young's modulus of the viscoelastic material is negligible compared to the elastic material.

As shown in figure the present model consists of two nodes and each node has four degrees of freedom. Nodal displacements are given by

$$\{\Delta^e\} = \{u_{1i}\ u_{3i}\ w_i\ \ \Phi_i\ u_{1j}\ u_{3j}\ w_j\ \Phi_j\} \tag{4.5}$$

Where i and j are elemental nodal numbers. The axial displacement of the constraining layer, the transverse displacement and the rotational angle, can be expressed in terms of nodal displacements and finite element shape functions.

$$u_1 =[N_1]\{\Delta^e\},\ u3=[N_3]\{\Delta^e\},\ w = [N_w]\{\Delta^e\},\ \Phi = [N_w]'\{\Delta^e\} \tag{4.6}$$

Where prime denotes differentiation with respect to axial co-ordinate x and the shape functions are given by

$$[N_1] = [1-\xi\ 0\ 0\ 0\ \xi\ 0\ 0\ 0]$$

$$[N_3] = [0\ 1-\xi\ 0\ 0\ 0\ \xi\ 0\ 0\ ]$$

And

$$[N_w] = [0\ 0\ (1-3\xi^2+2\xi^3)\ (\xi-2\xi^2+\xi^3)L_e\ 0\ 0\ \ 3\xi^2-2\xi^3\ (-\xi^2+\xi^3)L_e] \tag{4.7}$$

$\xi = x/L_e$ an d L_e is the length of the element.

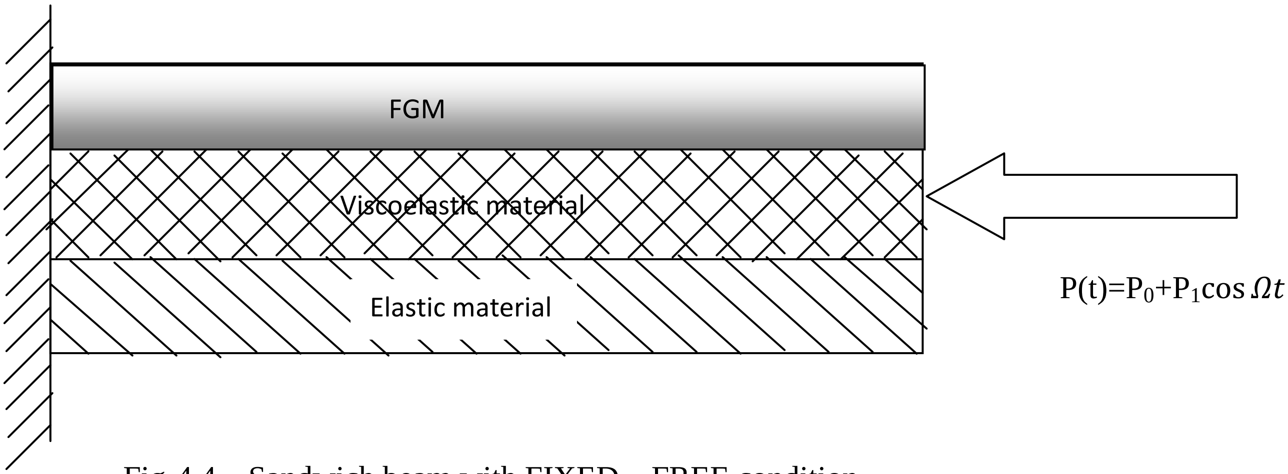


Fig-4.4 Sandwich beam with FIXED – FREE condition

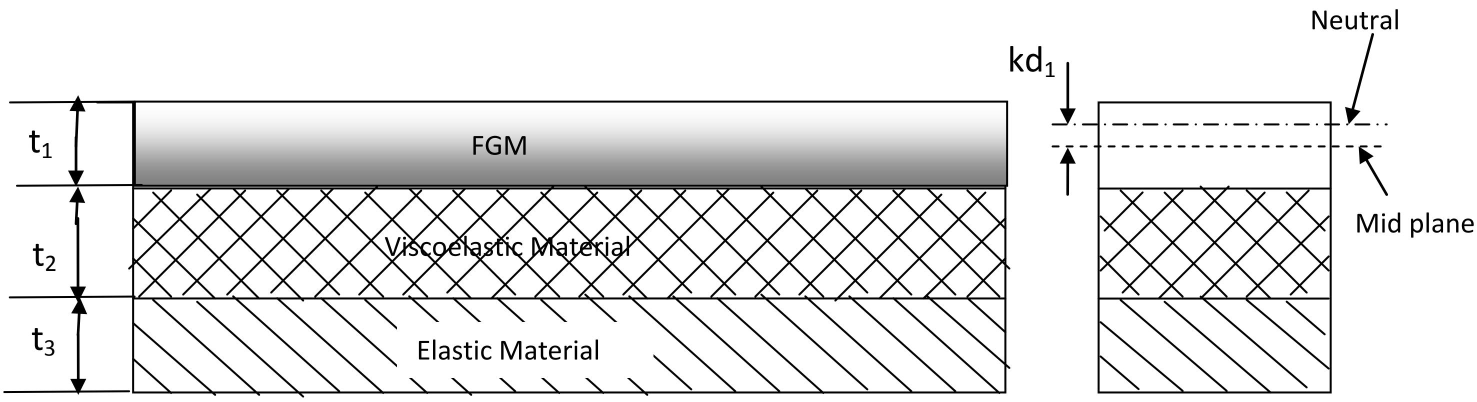


Fig-4.5 Position of neutral plane and mid plane of FGM constraining layer.

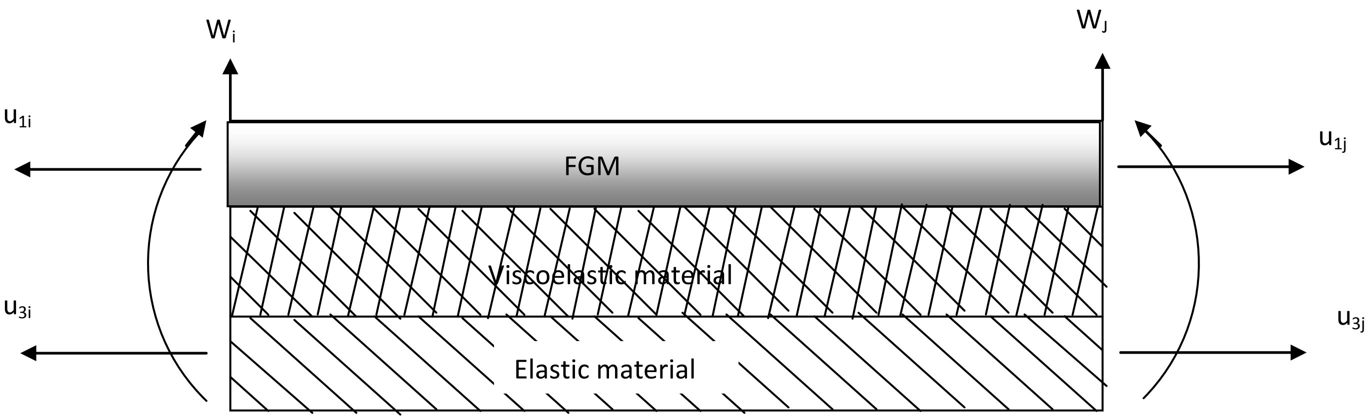


Fig-4.6 Sandwich beam finite element with DOF

4.2.1 Constraining layer

The potential energy of the constraining layer is expressed as

$$U_1^{(e)} = \frac{1}{2} I_1 \int_0^{L_e} E_1 \left[\frac{d^2w}{dx^2}\right]^2 dx + \frac{1}{2} \mathrm{A}_1 \int_0^{L_e} \mathrm{E}_1 \left[\frac{du}{dx}\right]^2 dx \tag{4.8}$$

Where $I_1 = b \int_{-\left(\frac{h_1}{2} - Kd_1\right)}^{\left(\frac{h_1}{2} - Kd_1\right)} z^2 \; dz$

$$\mathrm{A}_1 = b \int_{-\left(\frac{h_1}{2} - Kd_1\right)}^{\left(\frac{h_1}{2} - Kd_1\right)} dz$$

Where E, A, I are the Young's modulus, cross-sectional area and moment of Inertia respectively. The notation 1 represents the constraining layer.

The kinetic energy of the constraining layer is written as

$$T_1^{(e)} = \frac{1}{2} A_1 \int_0^{L_e} \rho_1 \left[\frac{dw}{dt}\right]^2 dx + \frac{1}{2} A_1 \int_0^{L_e} \rho_1 A_1 \left[\frac{du}{dt}\right]^2 dx \tag{4.9}$$

Where $\mathrm{A}_1 = b \int_{-\left(\frac{h_1}{2} - Kd_1\right)}^{\left(\frac{h_1}{2} - Kd_1\right)} dz$

4.2.2 Base layer:

The potential energy of the base layer is expressed as

$$U_3^{(e)} = \frac{1}{2} \int_0^{L_e} E_3 I_3 \left[\frac{d^2w}{dx^2}\right]^2 dx + \frac{1}{2} \int_0^{L_e} \mathrm{E}_3 \mathrm{A}_3 \left[\frac{du}{dx}\right]^2 dx \tag{4.10}$$

Where E, A, I are the Young's modulus, cross-sectional area and moment of Inertia of the base layer material respectively. The notation 3 represents the base layer.

The kinetic energy of the base layer is written as

$$T_3^{(e)} = \frac{1}{2} \int_0^{L_e} \rho_3 A_3 \left[\frac{dw}{dt}\right]^2 dx + \frac{1}{2} \int_0^{L_e} \rho_3 A_3 \left[\frac{du}{dt}\right]^2 dx \tag{4.11}$$

Where ρ is the mass density.

By submitting Eq.(4.6) into Eq.(4.8) Eq.(4.9), Eq.(4.10) and Eq.(4.11), the element potential energy and the kinetic energy of the constraining layer and Base layer can be written as

$$\mathrm{U}_k^{(e)} = \frac{1}{2} \{\Delta^{(e)}\}\left(\left[K_{ku}{}^{(e)}\right] + \left[K_{kw}{}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1,3 \tag{4.12}$$

And

$$T_k^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}\left(\left[M_{ku}{}^{(e)}\right] + \left[M_{kw}{}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k=1,3 \qquad (4.13)$$

Where

$$\left[K_{ku}{}^{(e)}\right]=\left[K_{1u}{}^{(e)}\right] + \left[K_{3u}{}^{(e)}\right]=E_1A_1\int_0^{L_e}[N_1]^T[N_1]\,dx+E_3A_3\int_0^{L_e}[N_3]^T[N_3]\,dx$$

$$\left[K_{kw}{}^{(e)}\right]=\left[K_{1w}{}^{(e)}\right]\left[K_{3w}{}^{(e)}\right]=E_1I_1\int_0^{L_e}[N_w]^T[N_w]\,dx+E_3I_3\int_0^{L_e}[N_w]^T[N_w]\,dx$$

$$\left[M_{ku}{}^{(e)}\right]=\left[M_{1u}{}^{(e)}\right] + \left[M_{3u}{}^{(e)}\right]=\rho_1A_1\int_0^{L_e}[N_1]^T[N_1]\,dx+\rho_3A_3\int_0^{L_e}[N_3]^T[N_3]\,dx$$

$$\left[M_{kw}{}^{(e)}\right]=\left[M_{1w}{}^{(e)}\right] + \left[M_{3w}{}^{(e)}\right]=\rho_1A_1\int_0^{L_e}[N_w]^T[N_w]\,dx+\rho_3A_3\int_0^{L_e}[N_w]^T[N_3]\,dx$$

4.2.3 Viscoelastic layer

The axial displacement u_v and γ_v shear strain of the viscoelastic layer is derived from kinematic relationships between the constraining layers as given by mead and Markus [28]. They are expressed as follows:

$$U_v = \frac{u_1+u_3}{2} + \frac{((t_1+2kd_1)-t_3)}{4}\frac{\partial w}{\partial x} \qquad (4.14)$$

$$\gamma_v = \frac{\partial w}{\partial x}\left[\frac{2t_2+(t_1+kd_1)+t_3}{2t_2}\right] + (u_1 - u_3) \qquad (4.15)$$

Substituting Eq. (4.6) into (4.14) and Eq.(4.15) γ_v and u_v can be expressed in terms of nodal displacements and element shape functions:

$$u_v = (N_v)\{\Delta^{(e)}\}$$

$$\gamma_v = (N_\gamma)\{\Delta^{(e)}\}$$

Where $(N_v) = \frac{1}{2}\left((N_1) + (N_3)\right)\frac{((t_1+2kd_1)-t_3)}{4}(N_w)$

$$(N_\gamma) = \frac{1}{2}\frac{((N_1)-(N_3))}{t_2} + \frac{((t_1+kd_1)+2t_2+t_3)}{t_2}(N_w)$$

The potential energy of the viscoelastic layer due to shear deformation is written as

$$U_v{}^{(e)} = \frac{1}{2}\int_0^{L_e} G_v A_v \gamma_v{}^2\, dx \qquad (4.16)$$

Where A_v is the cross-sectional area and G_v is the complex shear modulus of viscoelastic layer.

The kinetic energy of viscoelastic layer is written as

$$T_v{}^{(e)} = \frac{1}{2}\int_0^{L_e} \left\{\left(\frac{dw}{dt}\right)^2 + \left(\frac{du_v}{dt}\right)^2\right\} dx \qquad (4.17)$$

Substituting Eq (4.6) into Eqs.(4.16) and (4.17), the potential energy and kinetic energy of viscoelastic material is given by

$$U_v{}^{(\mathrm{e})} = \frac{1}{2}\{\Delta^{(e)}\}\left(K_v{}^{(e)}\right)\{\Delta^{(e)}\} \qquad (4.18)$$

$$T_v{}^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}\left(\left[M_v{}^{(e)}\right]\right)\{\dot{\Delta}^{(e)}\} \qquad (4.19)$$

Where $\left[K_v{}^{(e)}\right] = G_v A_v \int_0^{L_e} \left[N_\gamma\right]^T \left[N_\gamma\right] dx$

$$\left[M_v{}^{(e)}\right] = \rho_v A_v \int_0^{L_e} [N_v]^T [N_v]\, dx + \rho_v A_v \int_0^{L_e} [N_v]^T [N_v]\, dx$$

'·' denotes differentiation with respect to time t.

4.2.4 Work done by axial periodic force:

Work done by axial periodic force P(t) is written as

$$\mathrm{W_p}{}^{(\mathrm{e})} = \frac{1}{2}\int_0^{L_e} P(t)\left[\frac{dw}{dx}\right]^2 dx \qquad (4.20)$$

Substituting Eq.(4.6) into Eq.(4.20), the work done by the axial periodic load can be expressed as

$$\mathrm{W_p}{}^{(\mathrm{e})} = \frac{1}{2}\{\Delta^{(e)}\}^T P(t) k_p{}^{(e)}\{\Delta^{(e)}\} \qquad (4.21)$$

Where $\left[k_p{}^{(e)}\right] = \int_0^{L_e} [N_w]^T [N_w]\, dx$

The dynamic load P(t) is periodic and can be expressed in the form $P(t)=P_0+P_1\cos \Omega t$, where Ω is the disturbing frequency, P_0 the static and P_1 the amplitude of time dependent component of the load ,can be represented as the fraction of the fundamental static buckling load

$P_{cr}=\left(\frac{\pi^2 2EI}{L^2}\right)$ of a reference Euler beam, which is defined as having flexural rigidity 2EI and mass per unit length same as that of the original sandwich beam with pin-pin end conditions. Hence substituting P(t)=$\alpha P_{cr} + \beta P_{cr}\cos\Omega t$ with α and β as static and dynamic load factors respectively.

4.2.5 Equation of motion:

The element equations of motion for a sandwich beam with constrained damping layer subjected to an axial periodic load is derived by using extended Hamilton's principle.

$$\delta\int_{t_1}^{t_2}\left(T^{(e)} - U^{(e)} + W_p{}^{(e)}\right)dt = 0 \tag{4.22}$$

Substituting Eqn.(4.12),(4.13), (4.18), (4.19), and (4.21) in to Eq.(4.22) the element equation of motion for the sandwich beam element are obtained as follows:

$$\left[M^{(e)}\right]\left\{\ddot{\Delta}^{(e)}\right\}\left[K^{(e)}\right]\left\{\Delta^{(e)}\right\} - \beta P_{cr}\cos\Omega t\left[K_p{}^{(e)}\right]\left\{\Delta^{(e)}\right\} = 0 \tag{4.23}$$

Where

$$\left[M^{(e)}\right] = \left[M_{1u}{}^{(e)}\right] + \left[M_{1u}{}^{(e)}\right] + \left[M_{3u}{}^{(e)}\right] + \left[M_{3u}{}^{(e)}\right] + \left[M_v{}^{(e)}\right]$$

$$\left[K^{(e)}\right] = \left[K_{1u}{}^{(e)}\right] + \left[K_{1u}{}^{(e)}\right] + \left[K_{3u}{}^{(e)}\right] + \left[K_{3u}{}^{(e)}\right] + \left[K_{v\gamma}{}^{(e)}\right]$$

Assembling individual elements, the equation of motion of the global system can be written as

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} - P(t)\left[K_p\right]\{\Delta\} = 0 \tag{4.24}$$

By substituting P(t), the equation (3.22) becomes

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} - (P_0 + P_1\cos\Omega t)\left[K_p\right]\{\Delta\} = 0 \tag{4.25}$$

$$[M]\{\ddot{\Delta}\} + \left([K] - P_0\left[K_p\right]\{\Delta\} - P_1\cos\Omega t\right)\left[K_p\right]\{\Delta\} = 0 \tag{4.26}$$

$$[M]\{\ddot{\Delta}\} + [\bar{K}]\{\Delta\} - \beta P_{cr}\cos\Omega t\left[K_p\right]\{\Delta\} = 0 \tag{4.27}$$

Where

$$[\bar{K}] = [K] - P_0\left[K_p\right] \tag{4.28}$$

The nodal displacement matrix{Δ} can be assumed as

$$\{\Delta\} = [\Phi]\{\Gamma\} \tag{4.29}$$

Where $[\Phi]$ is the normalised modal matrix corresponding to

$$[M]\{\ddot{\Delta}\} + [\bar{K}]\{\Delta\} = 0 \tag{4.30}$$

And $\{\Gamma\}$ is a new set of generalised coordinates.

Substituting Eq. (4.29) in Eq.(4.27), Eq.(4.27) transforms to the following set of coupled Mathieu equations.

$$\Gamma_m + (w_m)^2\Gamma_m + \beta P_{cr}\cos\Omega t \sum_{n=1}^{N} b_{mn}\Gamma_n = 0 \qquad m=1,2....N, \tag{4.31}$$

Where $(w_m)^2$ are the distinct eigen values of $[M]^{-1}[\bar{K}]$ and b_{mn} are the elements of the complex matrix $[B] = -[\Phi]^{-1}[M]^{-1}[K_p][\Phi]$ and

$$w_m = w_{m.R} + iw_{m.I} \qquad b_{mn} = b_{mn.R} + Ib_{mn.I}$$

4.2.6 Regions of instability:

The boundaries of the regions of instability for simple and combination resonance are obtained by applying the following conditions [49] to the Eq. (4.31)

(A) **Simple resonance**

The boundaries of the instability regions are given by

$$\left|\frac{\Omega}{2\omega_0} - \overline{\omega_{\mu.R}}\right| < \frac{1}{4}\left[\frac{\beta^2(b^2{}_{\mu\mu.R}+b^2{}_{\mu\mu.I})}{\overline{\omega_{\mu.R}}^2} - 16\bar{\omega}_{\mu.I}{}^2\right]^{\frac{1}{2}} \qquad \mu=1,2............N \tag{4.32}$$

Where $\omega_0 = \sqrt{2E_1I_1/\rho_1A_1L^4}$, $\bar{\omega}_{\mu.R}=\omega_{\mu.R}/\omega_0$ and $\omega_{\mu.I} = \omega_{\mu.I}/\omega_0$

When damping is neglected, the regions of instability are given by

$$\left|\frac{\Omega}{2\omega_0} - \overline{\omega_{\mu.R}}\right| < \frac{1}{4}\left[\frac{\beta(b_{\mu\mu.R})}{\bar{\omega}_{\mu.R}}\right] \qquad \mu=1,2..................N \tag{4.33}$$

(B)**Combination resonance of sum type**

The boundaries of the regions of instability of sum type are given by

$$\left|\frac{\Omega}{2\omega_0} - \frac{1}{2}(\bar{\omega}_{\mu.R} + \bar{\omega}_{v.R})\right| < \frac{1}{8}\frac{(\bar{\omega}_{\mu.I}+\bar{\omega}_{v.I})}{(\bar{\omega}_{\mu.I}\bar{\omega}_{v.I})^{1/2}}\left[\frac{\beta^2(b_{\mu.v.R}b_{v.\mu.R}+b_{\mu.v.I}b_{v.\mu.I})}{\omega_{\mu.R}\omega_{v.R}} - 16\bar{\omega}_{\mu.I}\bar{\omega}_{v.I}\right]^{1/2} \tag{4.34}$$

μ≠ ν, μ, $\nu = 1,2, \ldots \ldots \ldots \ldots \ldots \ldots \ldots . N$

When damping is neglected, the unstable regions are given by

$$\left|\frac{\Omega}{2\omega_0} - \frac{1}{2}\left(\overline{\omega}_{\mu.R} + \overline{\omega}_{\nu.R}\right)\right| < \frac{1}{4}\left[\frac{\beta^2\left(b_{\mu.\nu.R} + b_{\nu.\mu.I}\right)}{\omega_{\mu.R}\omega_{\nu.R}}\right]^{1/2} \quad (4.35)$$

μ≠ ν, μ, $\nu = 1,2, \ldots \ldots \ldots \ldots \ldots N$

(C) Combination resonance of difference type

The boundaries of the regions of difference type are given by

$$\left|\frac{\Omega}{2\omega_0} - \frac{1}{2}\left(\overline{\omega}_{\mu.R} - \overline{\omega}_{\nu.R}\right)\right| < \frac{1}{8}\frac{\left(\overline{\omega}_{\mu.I} + \overline{\omega}_{\nu.I}\right)}{\left(\overline{\omega}_{\mu.I}\overline{\omega}_{\nu.I}\right)^{1/2}}\left[\frac{\beta^2\left(b_{\mu.\nu.R}b_{\nu.\mu.R} - b_{\mu.\nu.I}b_{\nu.\mu.I}\right)}{\omega_{\mu.R}\omega_{\nu.R}} - 16\overline{\omega}_{\mu.I}\overline{\omega}_{\nu.I}\right]^{1/2} \quad (4.36)$$

μ $> \nu$, μ, ν = 1,2,.............................N

When damping is neglected, the unstable regions are given by

$$\left|\frac{\Omega}{2\omega_0} - \frac{1}{2}\left(\overline{\omega}_{\mu.R} - \overline{\omega}_{\nu.R}\right)\right| < \frac{1}{4}\left[\frac{\beta^2\left(b_{\mu.\nu.R}b_{\nu.\mu.I}\right)}{\omega_{\mu.R}\omega_{\nu.R}}\right]^{1/2} \quad (4.37)$$

μ $> \nu$, μ, ν = 1,2,.............................N

RESULTS and DISCUSSION

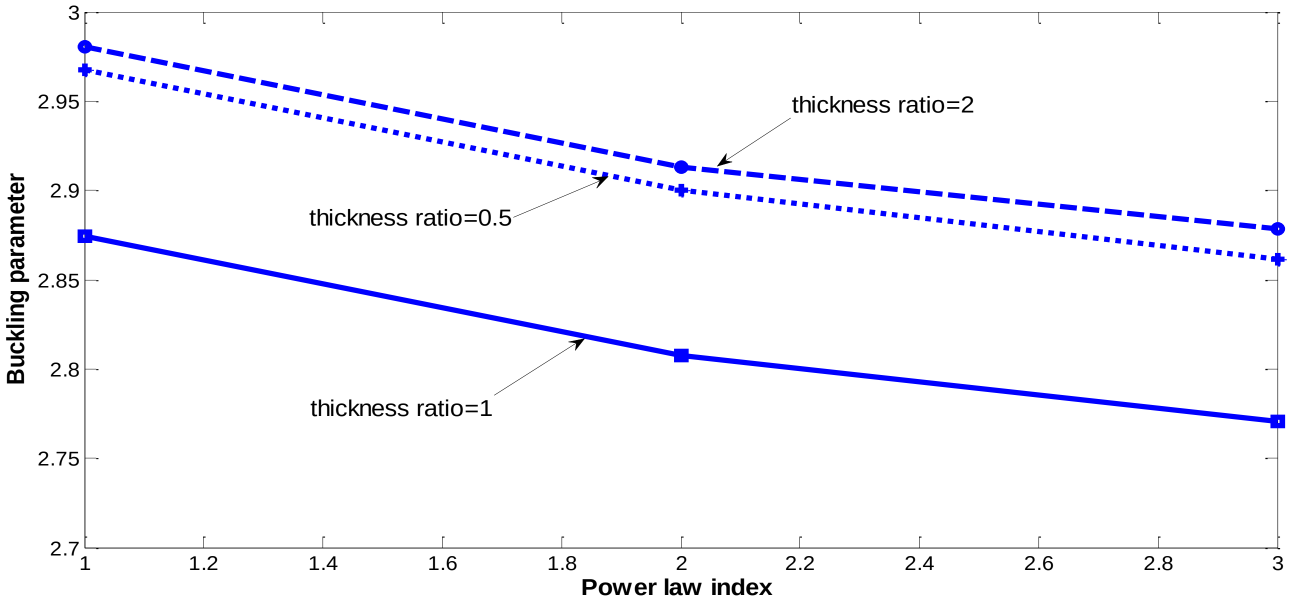


Figure-5.7 Effect of core thickness parameter(t_2/t_1) on the buckling load parametr(P_b) for core loss Factor(η_c)=0.3 and power law index(n)=1, 2, 3. First mode of vibration.

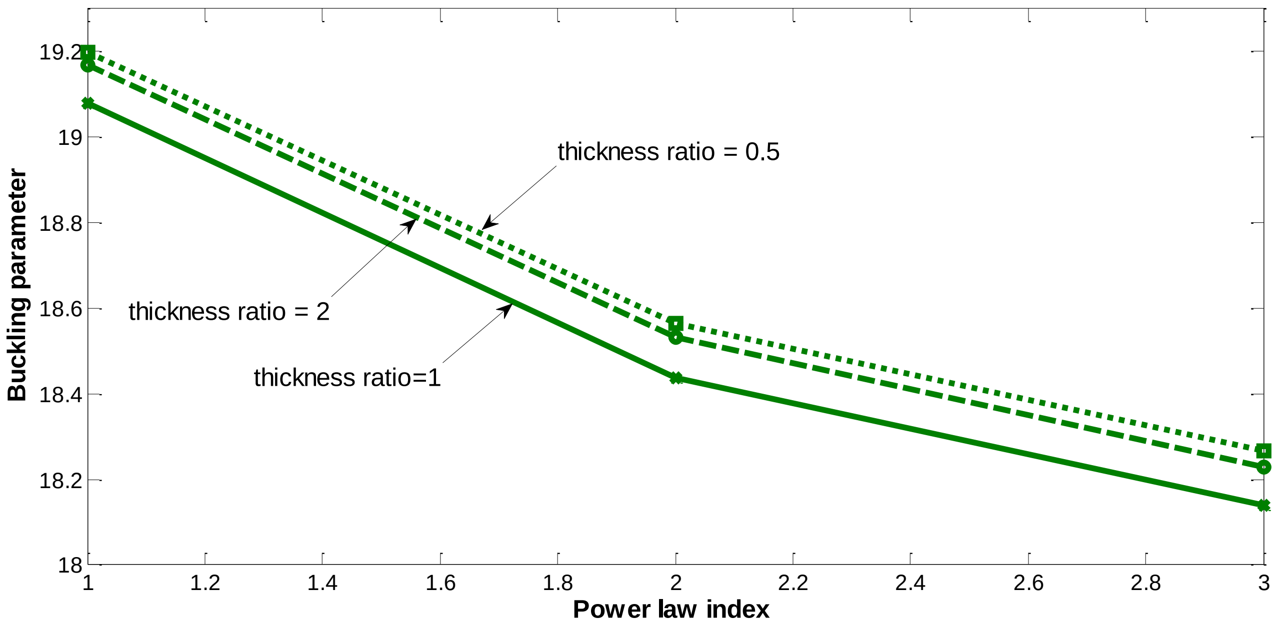


Figure- 5.8 Effect of core thickness parameter(t_2/t_1) on the buckling load parametr(P_b) for core loss factor(η_c)=0.3 and power law index(n)=1, 2, 3. Second mode of vibration.

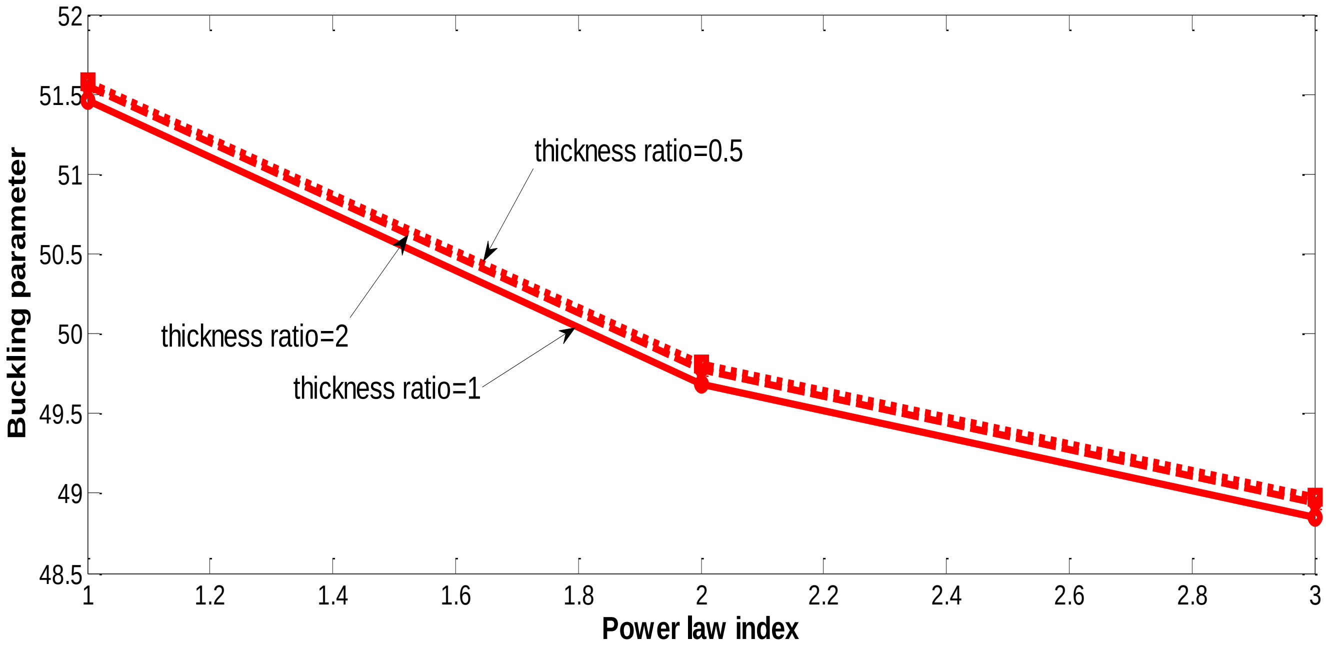


Figure- 5.9 Effect of core thickness parameter(t_2/t_1) on the buckling load parametr(P_b) for core loss Factor(η_c)=0.3 and power law index(n)=1, 2, 3. Third mode of vibration.

Figures 5.7 to 5.9 show first, second and third mode variation of buckling load parameter (P_b) with core thickness parameter(t_2/t_1) for different values of power law index i.e., 1, 2, & 3. Core loss factor is 0.3. buckling load parameter is defined as the ratio of critical buckling load of the beam to critical buckling load of a equivalent sandwich beam with steel base and alumina constraining layer.

With increase in power law index from 1 to 3, buckling load parameter decreases for all thickness ratios. Core thickness ratio(t_2/t_1) value 0.5 gives the highest buckling load and t_2/t_1=1 gives the lowest value.t_2/t_1=2 is in between these two.

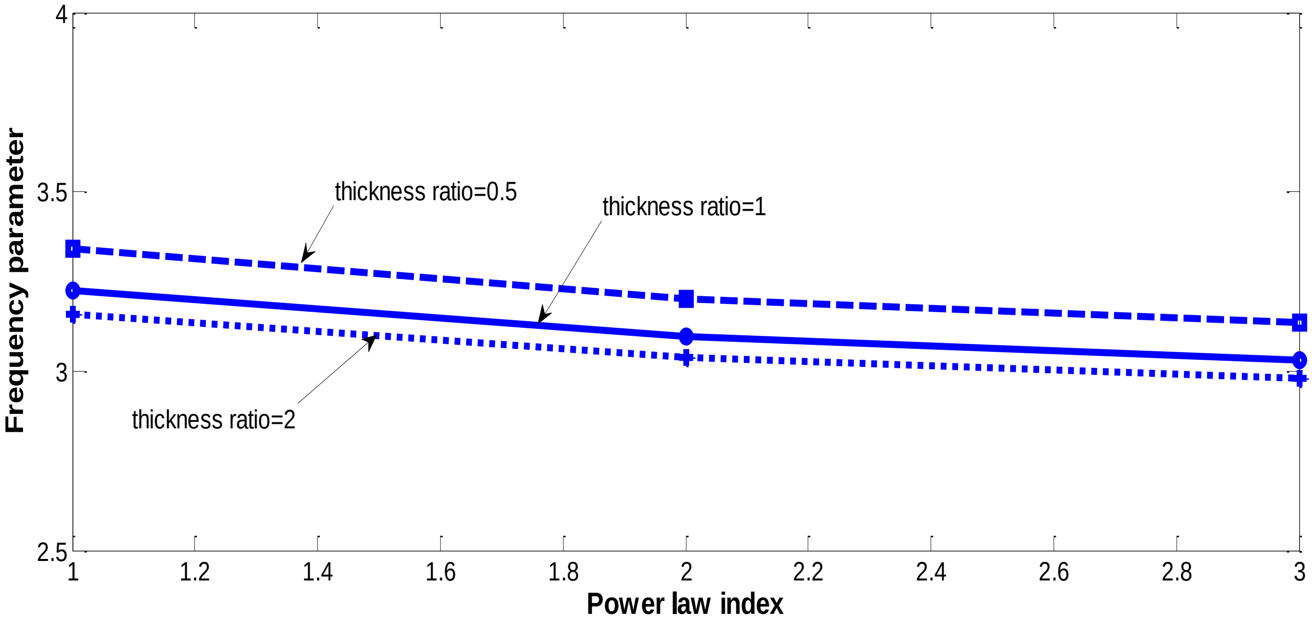


Figure-5.10 Effect of core thickness parameter(t_2/t_1) on the frequency parameter(f) for core loss factor(η_c)=0.3 and power law index(n)=1, 2, 3. First mode of vibration.

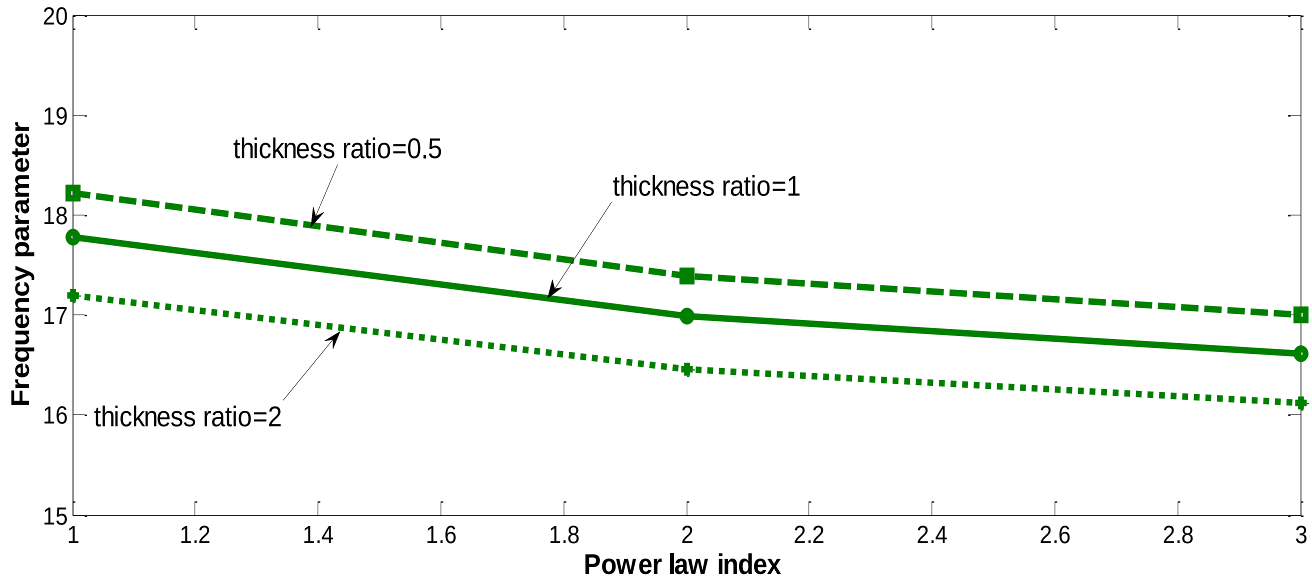


Figure-5.11 Effect of core thickness parameter(t_2/t_1) on the frequency parameter(f) for core loss factor(η_c)=0.3 and power law index(n)=1, 2, 3. Second mode of vibration.

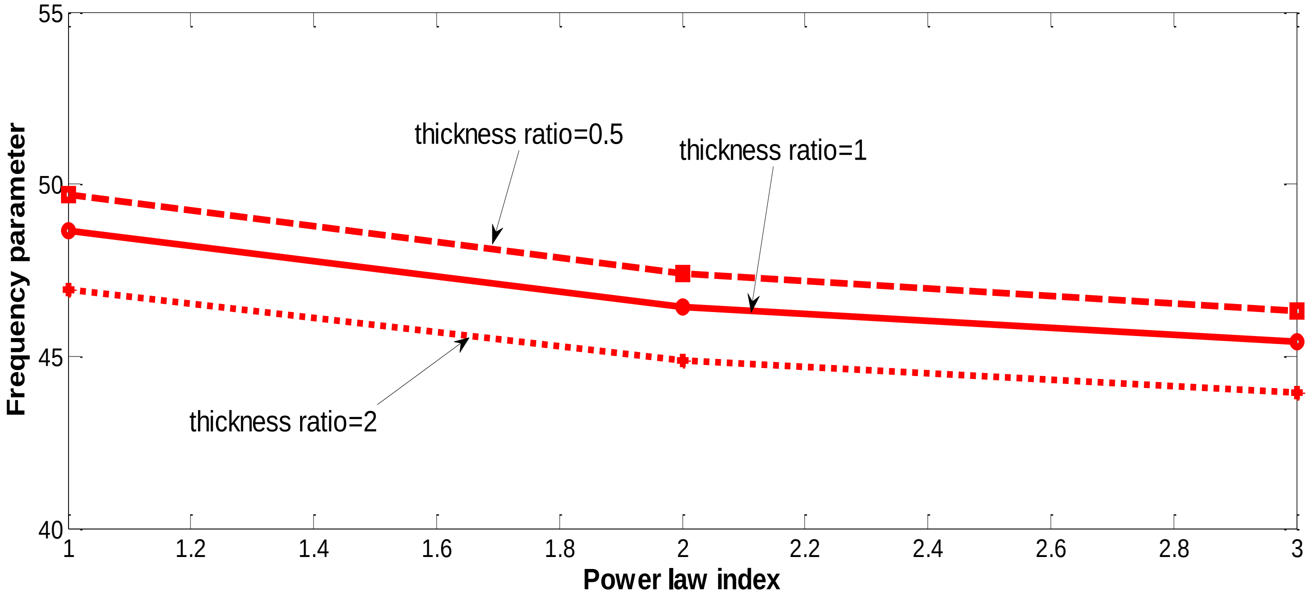


Figure-5.12 Effect of core thickness parameter(t_2/t_1) on the frequency parameter(f) for core loss factor(η_c)=0.3 and power law index(n)=1, 2, 3. Third mode of vibration.

Figures 5.10 to 5.12 show first, second and third mode variation of frequency parameter (f) with power law index for different values of core thickness Parameter(t_2/t_1). Core loss factor is 0.3. Frequency parameterr is defined as the ratio of frequency of the sandwich beam to natural frequency of a equivalent sandwich beam with steel base and alumina constraining layer.

With increase in power law index from 1 to 3, frequency parameter decreases for all thickness ratios. Core Thickness ratio value 0.5 gives the highest frequency and $t_2/t_1=2$ gives the lowest value.$t_2/t_1=1$ is in between 1 and 3.

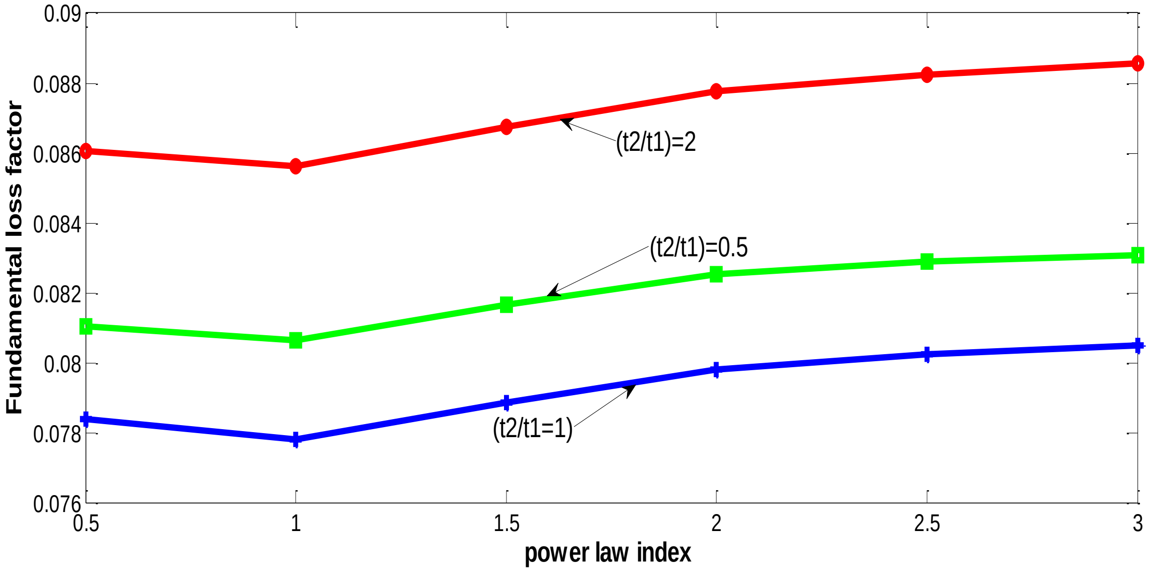


Figure- 5.13 Effect of core thickness parameter(t_2/t_1) on fundamental loss factor for core loss factor(η_c)=0.3 and power law index(n)=1, 2, 3. First mode case

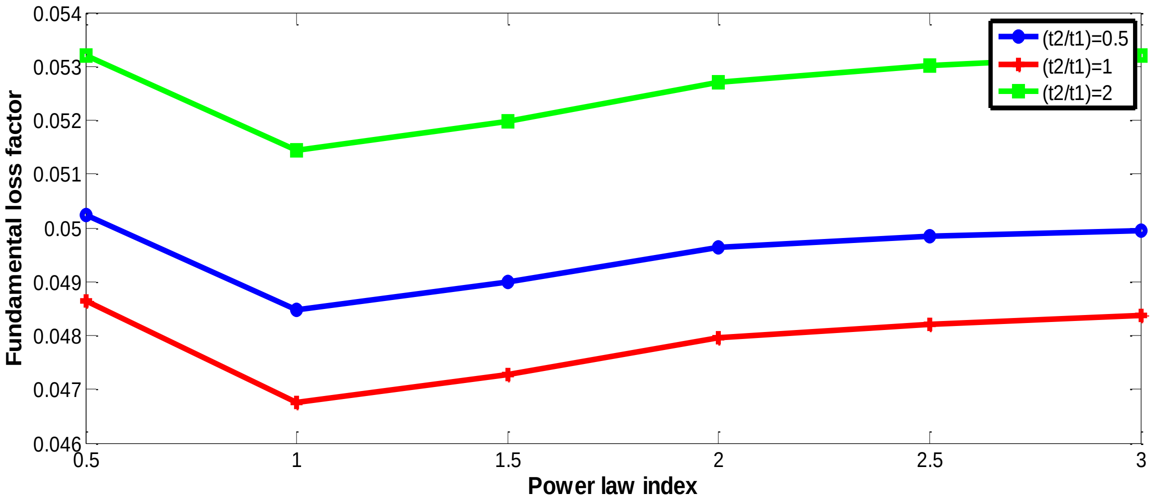


Figure-5.14 Effect of core thickness parameter(t_2/t_1) on the fundamental loss factor1 for core loss Factor(η_c)=0.3 and Power law index(n)=1, 2, 3. First mode case

The above figures 5.13 & 5.14 gives the varaition of the first fundamental loss factor for different power law index values, thickness ratio values & Core loss factorvalues of 0.3, 0.18. With increase in power law index value, first fundamental frequency decrases at first and then increases for both core loss factors 0.3 & 0.18. In the both case thickness ratio = 2 gives highest Fundamental loss factor value and t_2/t_1=1 gives the lowest value.Core loss factor 0.3 gives the high value of first fundamental loss factor value than 0.18.

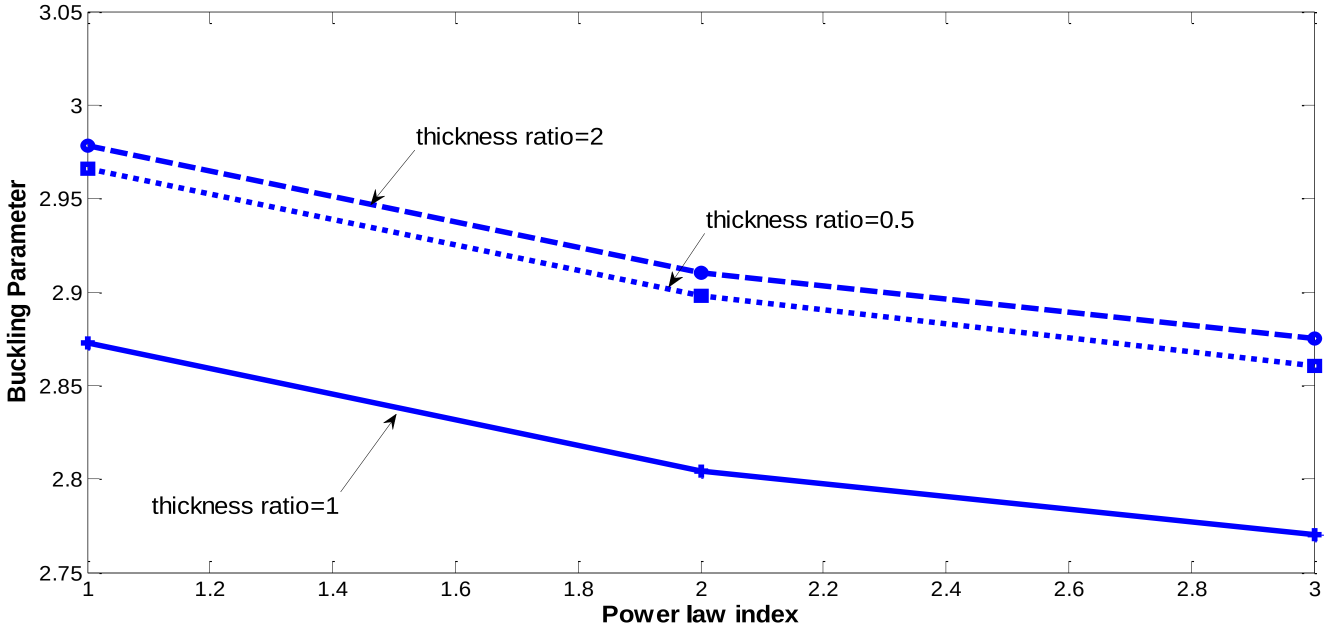


Figure-5.15 Effect of core thickness parameter(t_2/t_1) on the buckling load parametr(P_b) for core loss factor(η_c)=0.18 and power law index(n)=1, 2, 3. First mode of vibration.

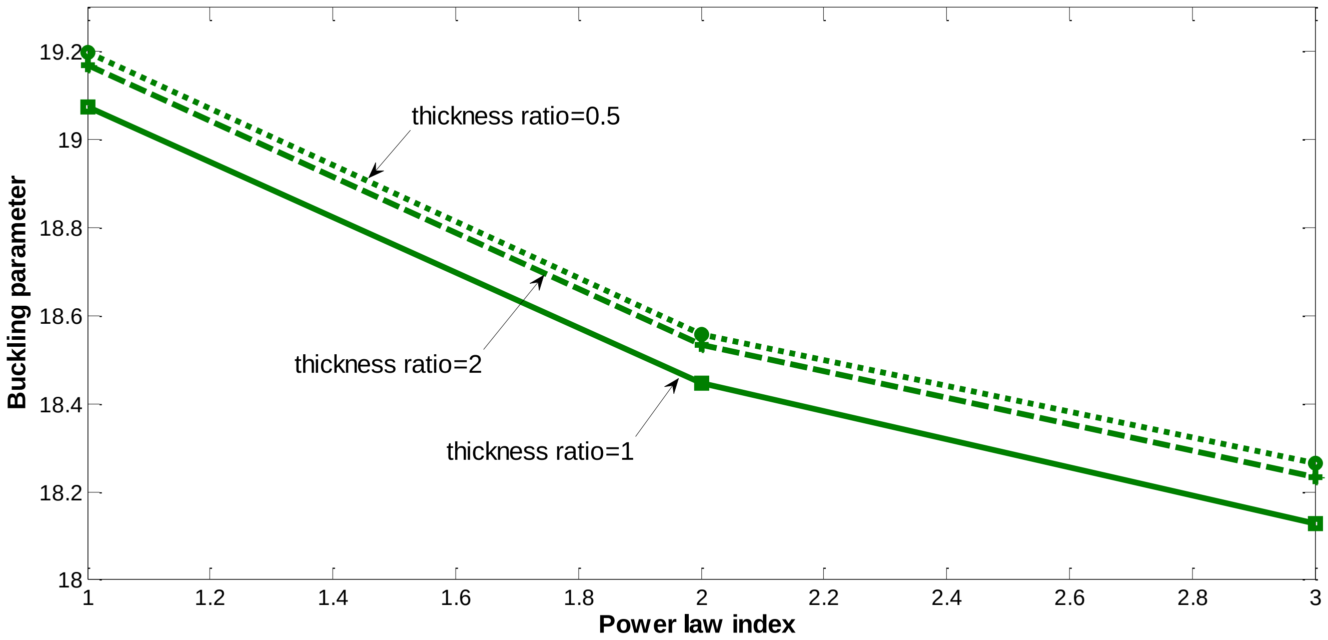


Figure-5.16 Effect of core thickness parameter(t_2/t_1) on the buckling load parametr(P_b) for core loss factor(η_c)=0.18 and power law index(n)=1, 2, 3. Second mode of vibration .

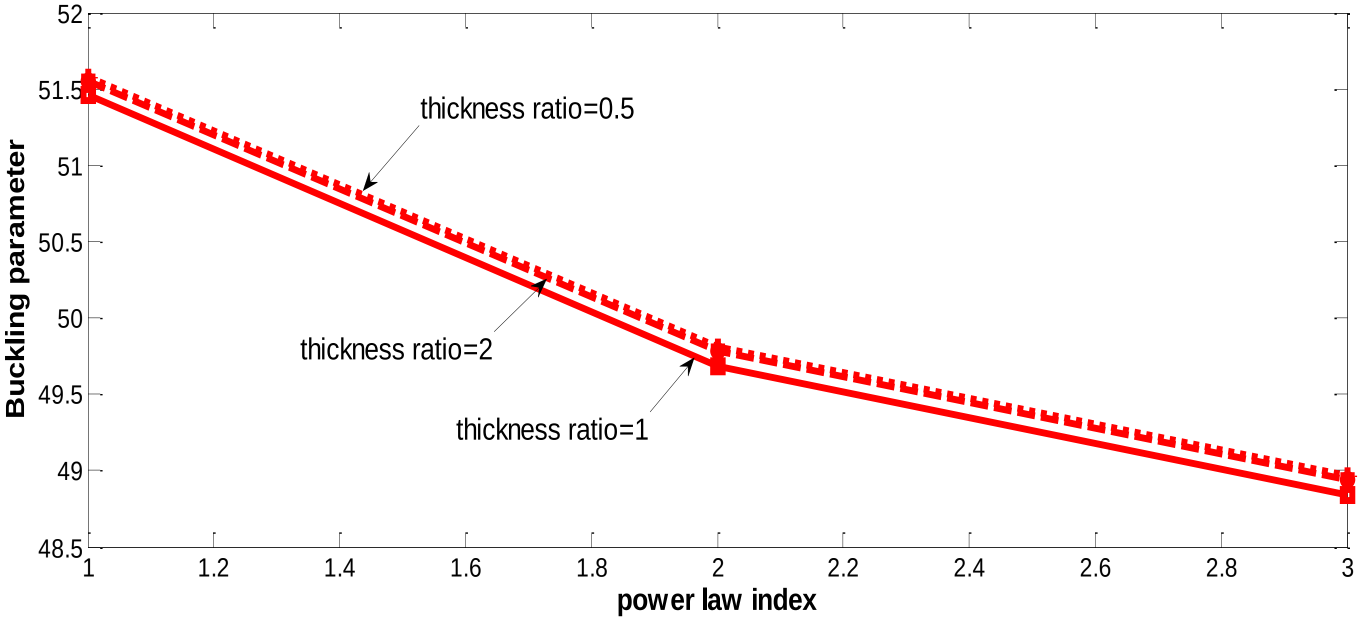


Figure-5.17 Effect of core thickness parameter(t_2/t_1) on the buckling load parametr(P_b) for core loss factor(η_c)=0.18 and power law index(n)=1, 2, 3. Third mode of vibration.

Figures 5.15 to 5.17 shows first, second and third mode variation of the buckling load parameter (P_b) with core thickness parameter(t_2/t_1) for different values of power law index i.e., 1, 2, & 3. Core loss factor is 0.18.

With increase in power law index from 1 to 3, buckling load parameter decreases for all thickness ratios. Core thickness ratio value 0.5 gives the highest buckling load and t_2/t_1=1 gives the lowest value.t_2/t_1=2 is in between these two.

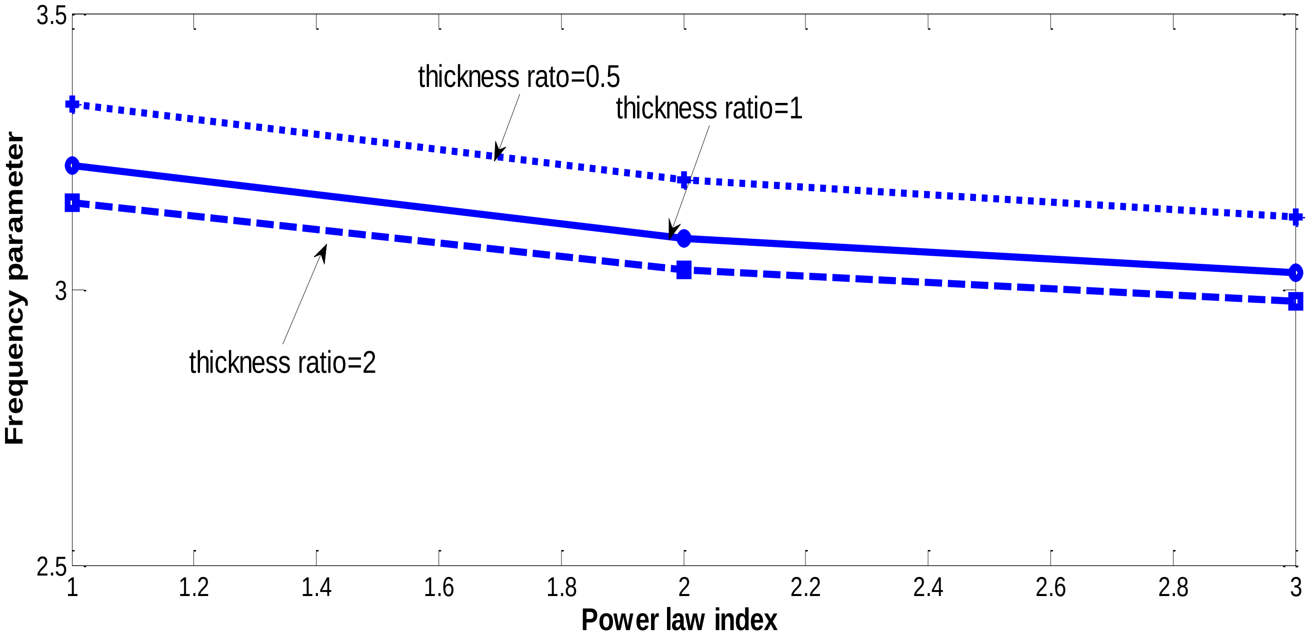


Figure-5.18 Effect of Core thickness parameter(t_2/t_1) on the frequency parameter(f) for core loss factor(η_c)=0.18 and power law index(n)=1, 2, 3. First mode of vibration.

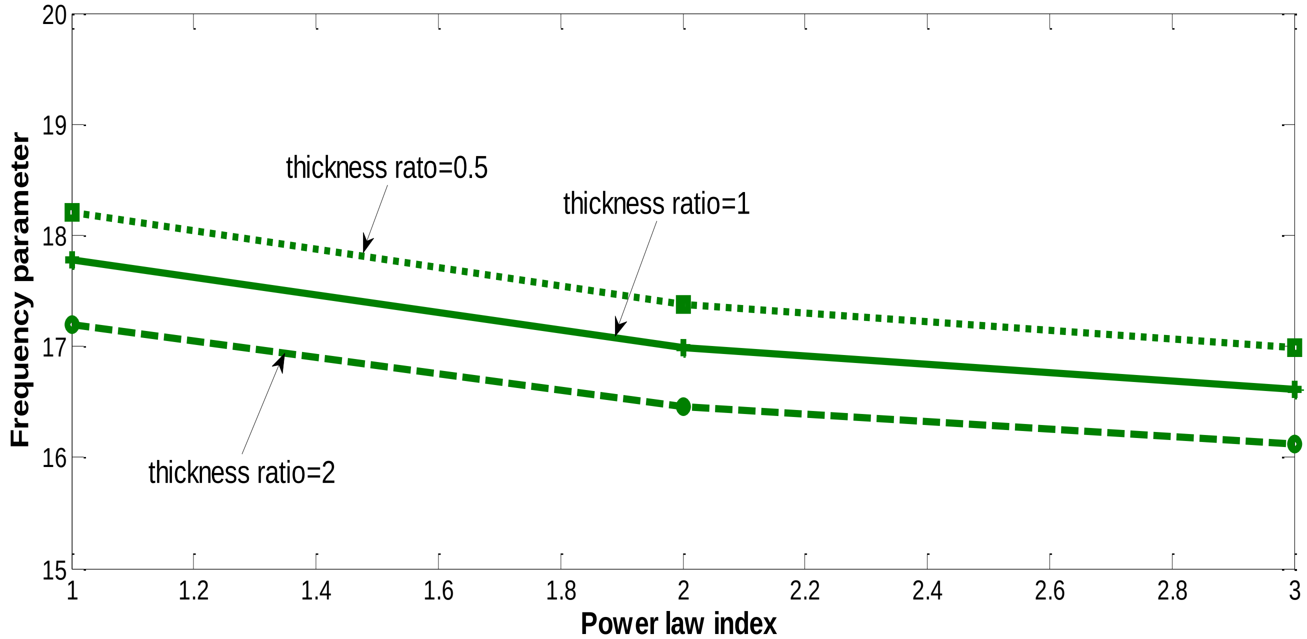


Figure-5.19 Effect of core thickness parameter(t_2/t_1) on the frequency parameter(f) for core loss Factor(η_c)=0.18 and Power law index(n)=1, 2, 3. Second mode of vibration.

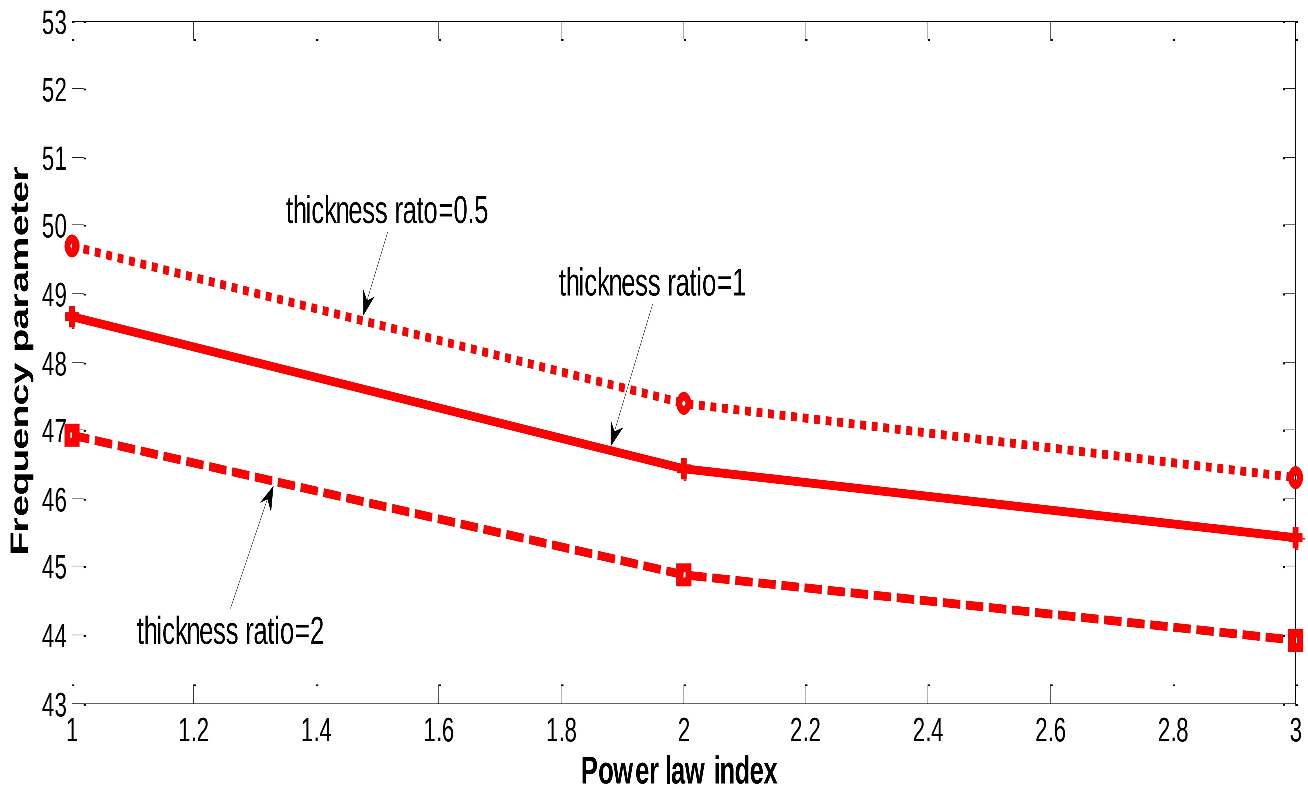


Figure-5.20 Effect of core thickness parameter(t_2/t_1) on the frequency parameter(f) for core loss factor(η_c)=0.18 and power law index(n)=1, 2, 3. Third mode of vibration.

Figures 5.18 to 5.20 show the variation of first, second and third mode of frequency parameter (f) with power law indexfor different values core thickness parameter(t_2/t_1) of i.e., 0.5, 1 & 2. Core loss factor is 0.18.

With increase in power law index from 1 to 3, frequency parameter decreases for all thickness ratios. Core thickness ratio value 0.5 gives the highest frequency and t_2/t_1=2 gives the lowest value.t_2/t_1=1 is in between these two.

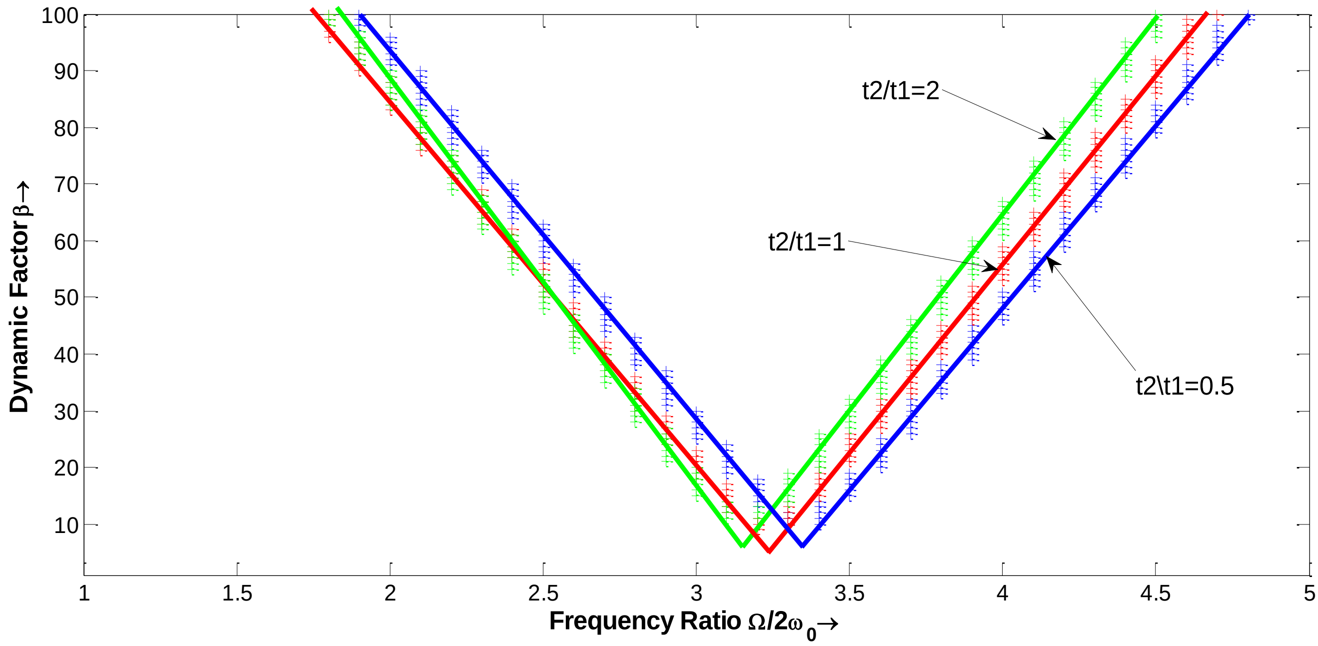


Figure-5.21 Instability Regions: power law index(n)=1, core loss factor(η_c)=0.3, core thickness parameter(t_2/t_1) = 0.5, 1, 2. First mode of vibration for simple resonance case.

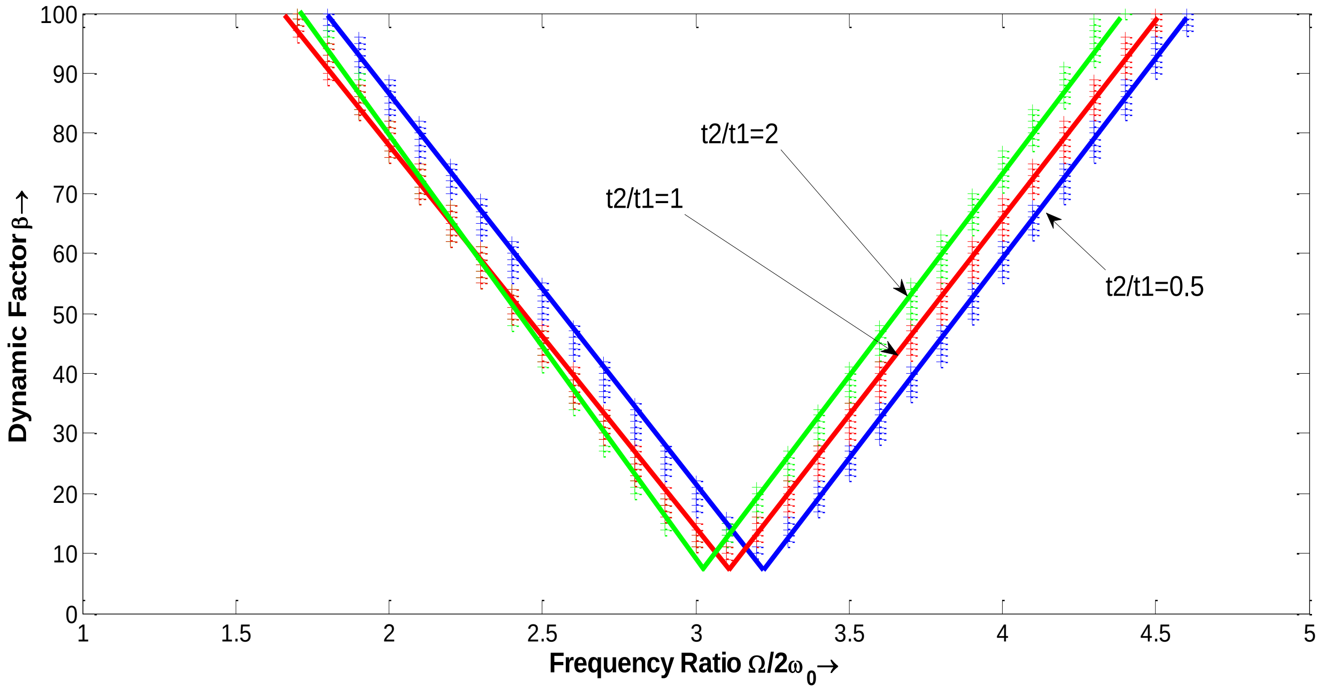


Figure-5.22 Instability Regions: Power law index(n)=2, core loss factor(η_c)=0.3, core thickness parameter(t_2/t_1) = 0.5, 1, 2. First mode of vibration for simple resonance case.

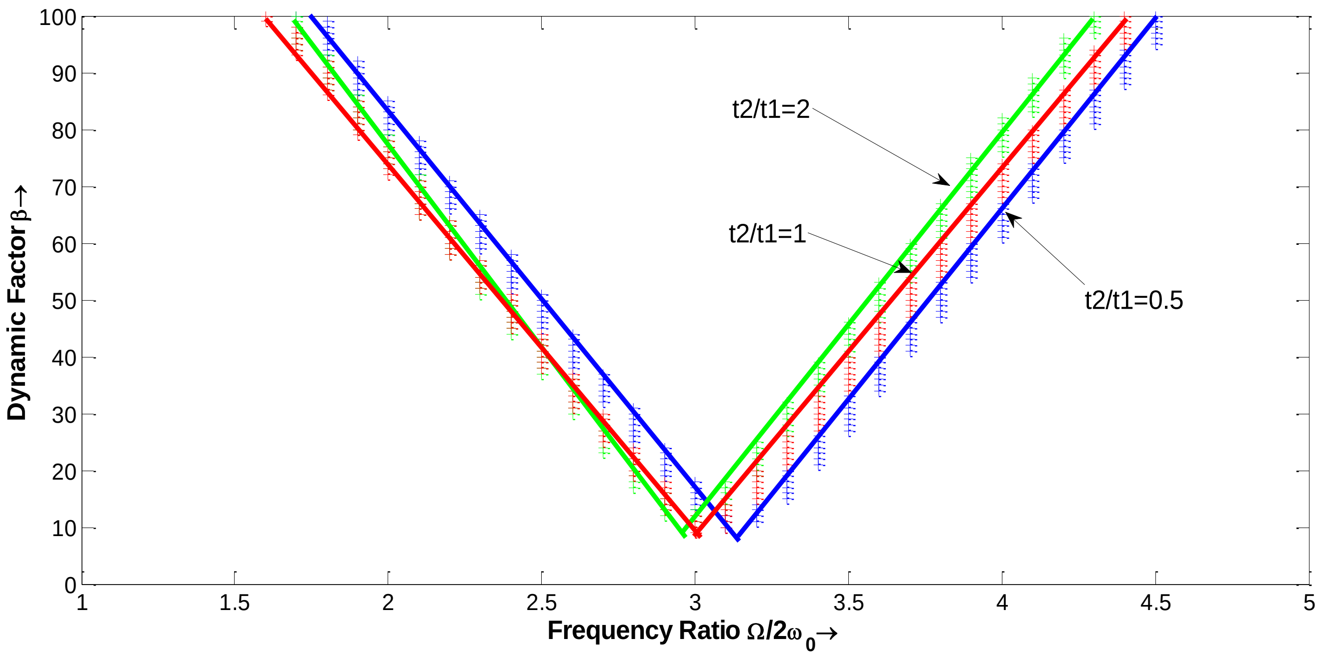


Figure-5.23 Instability Regions: Power law index(n)=3, core loss factor(η_c)=0.3, core thickness parameter(t_2/t_1) = 0.5, 1, 2.First mode of vibration and simple resonance case.

Figures from 5.21 to 5.23 gives the boundaries of stable and unstable regions of sandwich beam for first mode of vibration and simple resonance case with different values of thickness ratios.Power law index value is constant for a particular plot.

All plots follow the same pattern of variation of swiching of boundaries towards lower frequncy of excitation for thickness ratio values 0.5, 1 and 2 respectively.

Increase in the value of Power law index values from 1 to 3 is also switches the instability boundaries towards lower frequency of excitation which revials more probability of instability of beam.

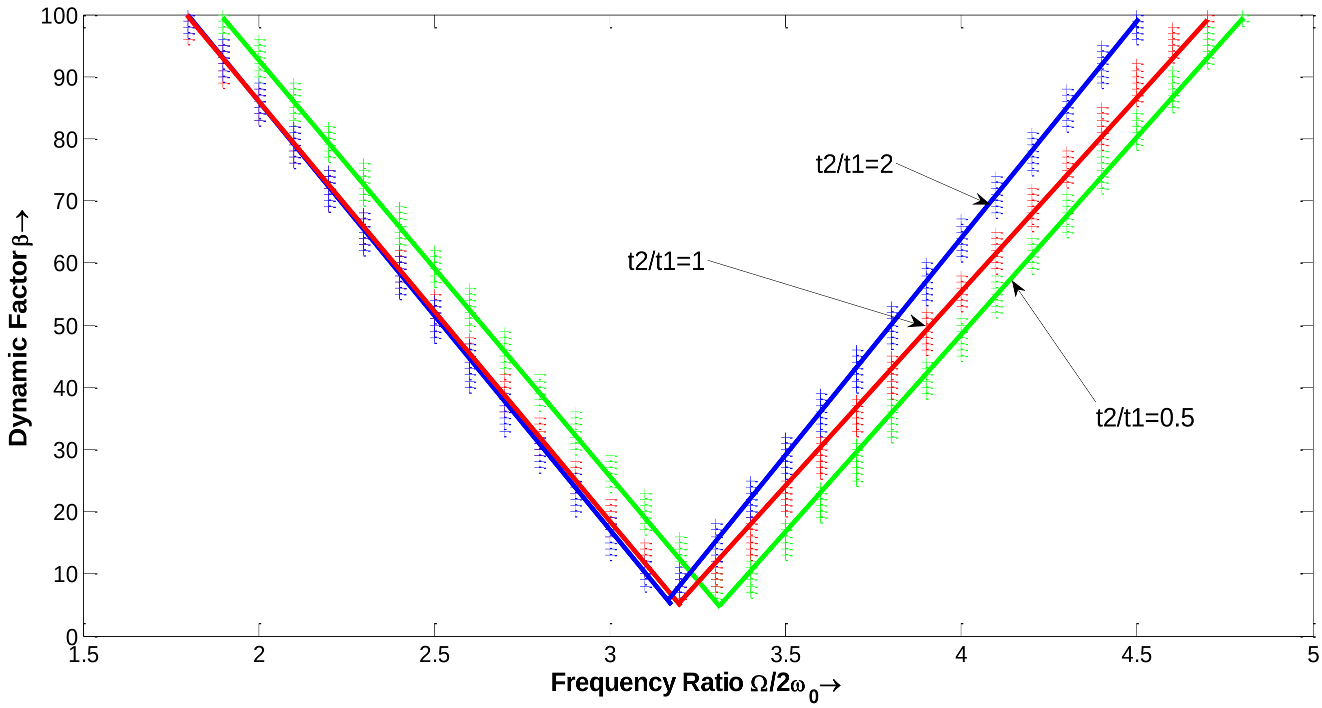


Figure-5.24 Instability Regions: Power law index(n)=1, core loss factor(η_c)=0.18,core thickness parameter(t_2/t_1) = 0.5, 1, 2. First mode of vibration and simple resonance case.

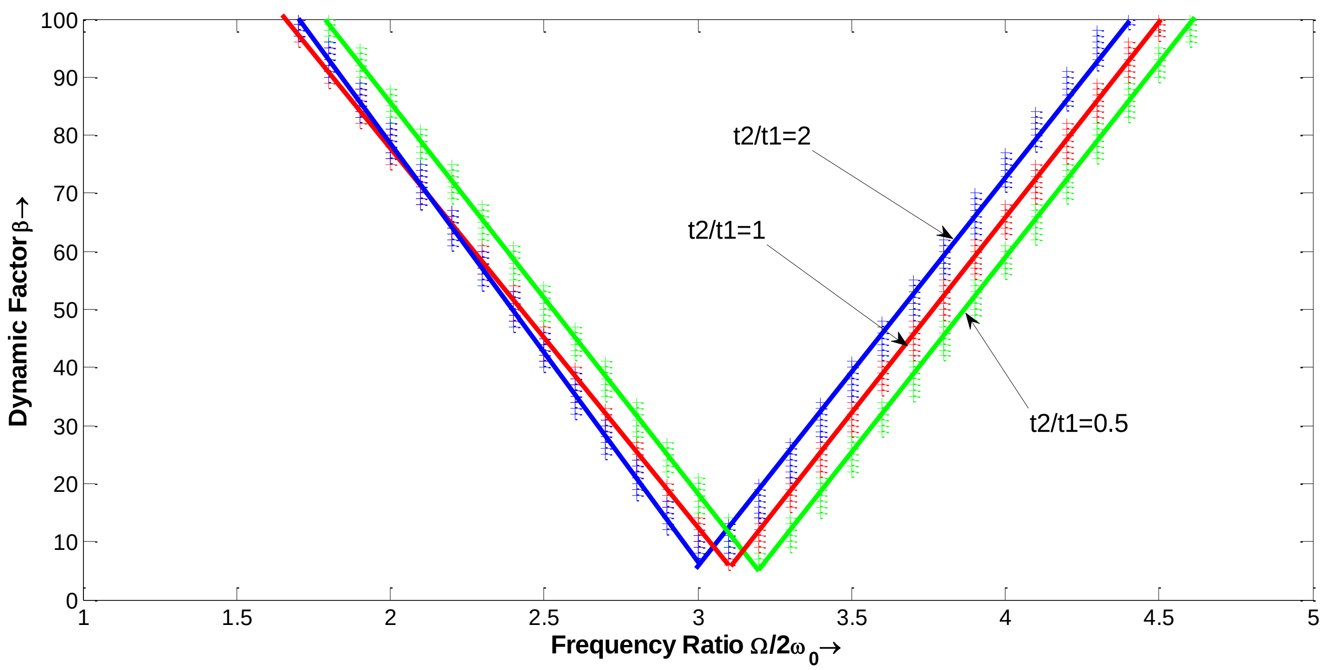


Figure-5.25 Instability Regions: power law index(n)=2, core loss factor(η_c)=0.18, core thickness parameter(t_2/t_1) = 0.5, 1, 2. First mode of vibration and Simple resonance case.

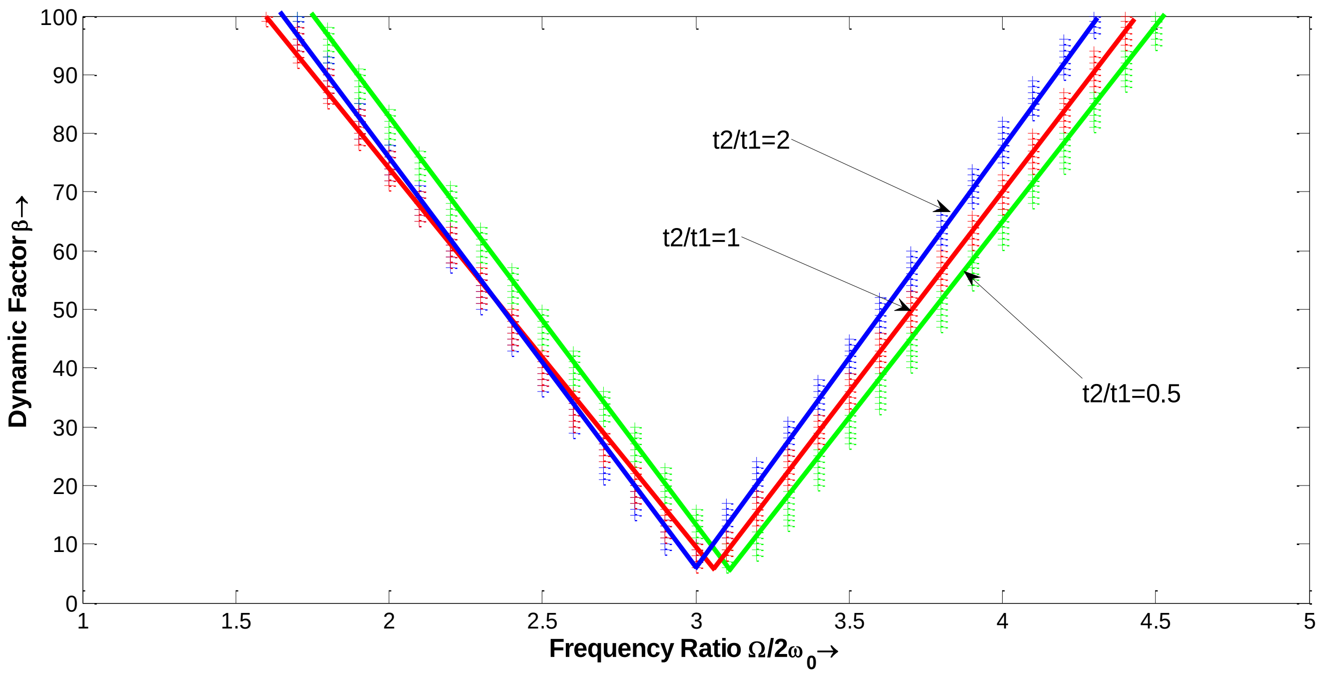


Figure-5.26 Instability Regions: Power law index(n)=3, core loss factor(η_c)=0.18, core thickness parameter(t_2/t_1) = 0.5, 1, 2. First mode of vibration and simple resonance case.

Figures from 5.24 to 5.26 gives the boundaries of stable and unstable regions of sandwich beam for first mode of vibration and Simple resonance case with different values of thickness ratios. Power law index value is constant for particular plot. For all plots Core loss factor is 0.18.All plots follow the same pattern of variation of swiching of boundaries towards lower frequcny of excitation for thickness ratio values 0.5, 1 and 2 respectively. Which results enhances instability of beam

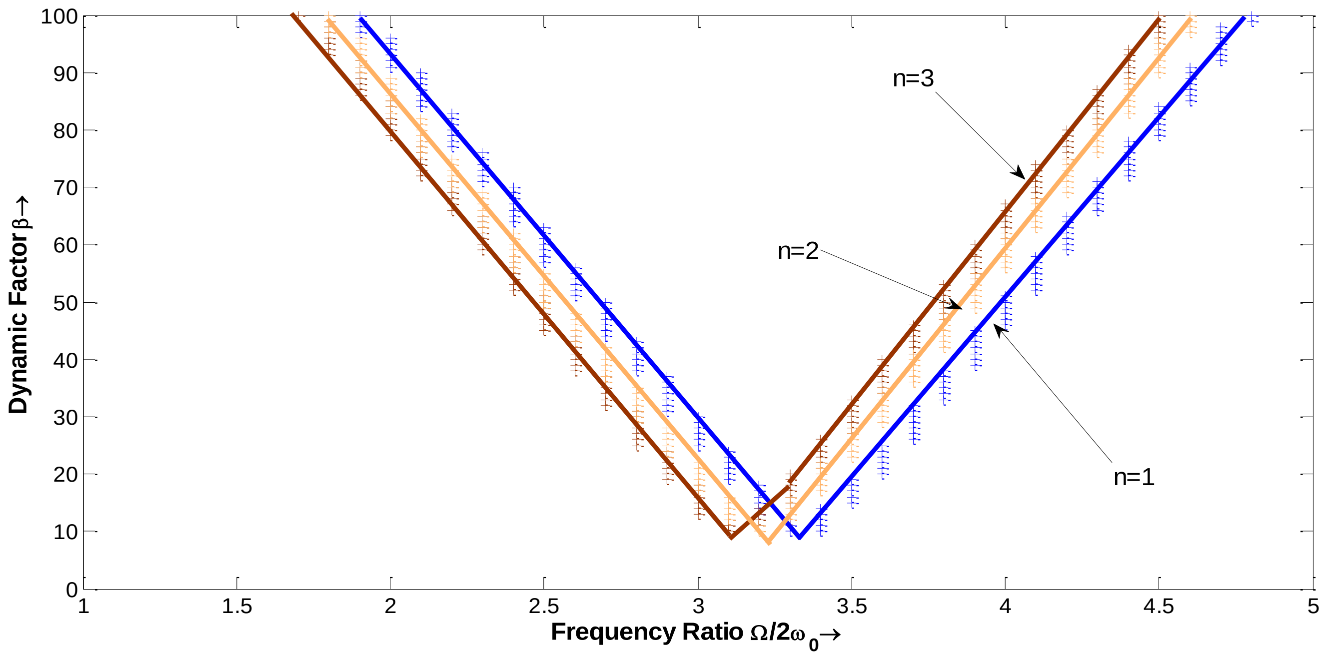


Figure-5.27 Instability Regions: Core loss factor(η_c)=0.3, Power law index(n)=1, 2, 3 Core thickness parameter(t_2/t_1) = 0.5, First mode of vibration and simple resonance case.

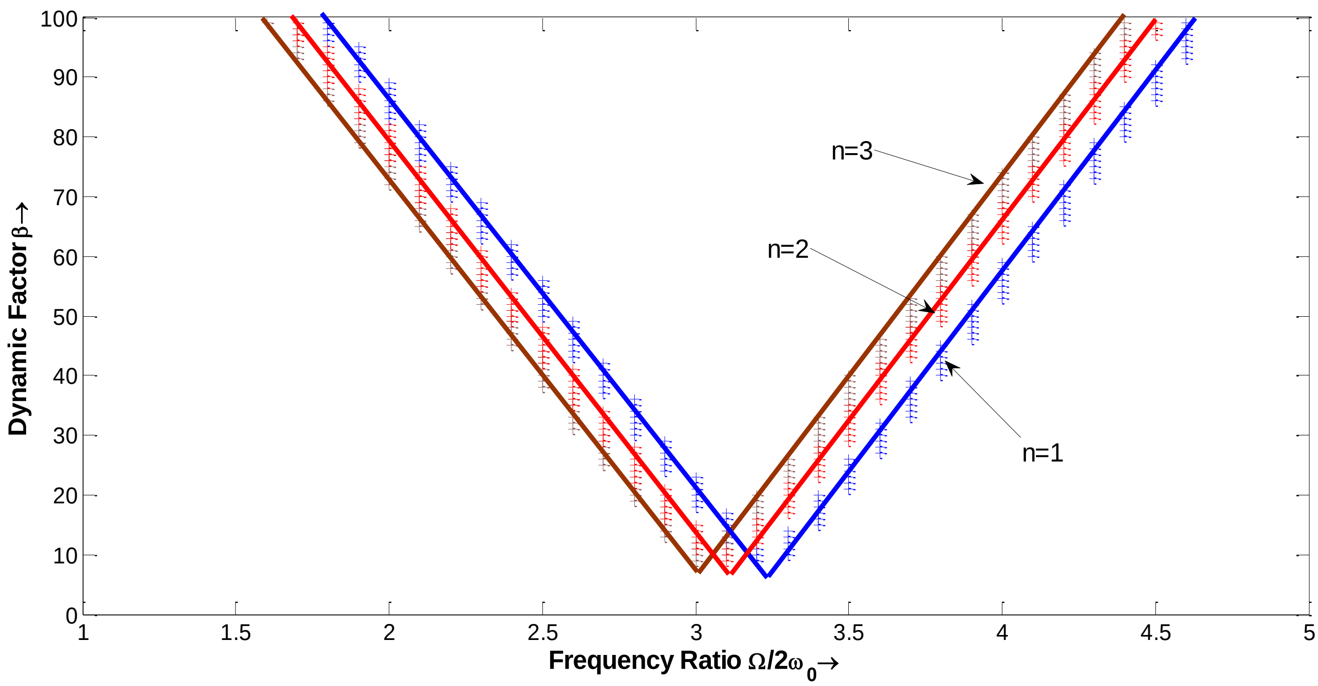


Figure-5.28 Instability Regions: Core loss factor(η_c)=0.3,power law index(n)=1, 2, 3 core thickness parameter(t_2/t_1) =1. First mode of vibration and simple resonance case.

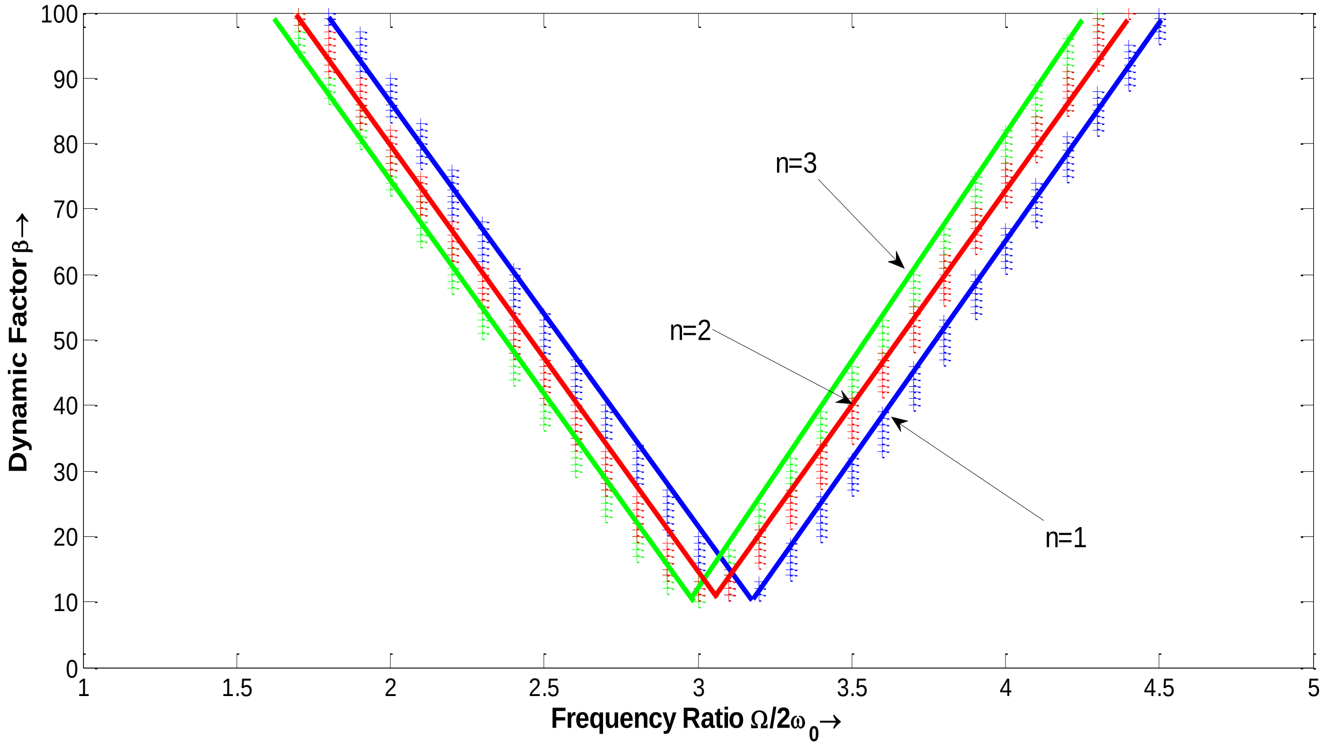


Figure-5.29 Instability Regions: core loss factor(η_c)=0.3,power law index(n)=1, 2, 3 core thickness parameter(t_2/t_1) = 2. First mode of vibration and simple resonance case.

Figures from 5.27 to 5.29 gives the boundaries of stable and unstable regions of sandwich beam for first mode of vibration with power law index values 1, 2 and 3. Thickness ratio is constant for particular plot.

All plots follow same pattern of variation of swiching of boundaries towards y-axis i.e., lower frequncy of excitation for Power law index values 1, 2 and 3.

Increase in the value of thickness ratio from 0.5 to 2 is also switches the instability boundaries towards lower frequency of excitation which exhibits more probability of instability of beam.

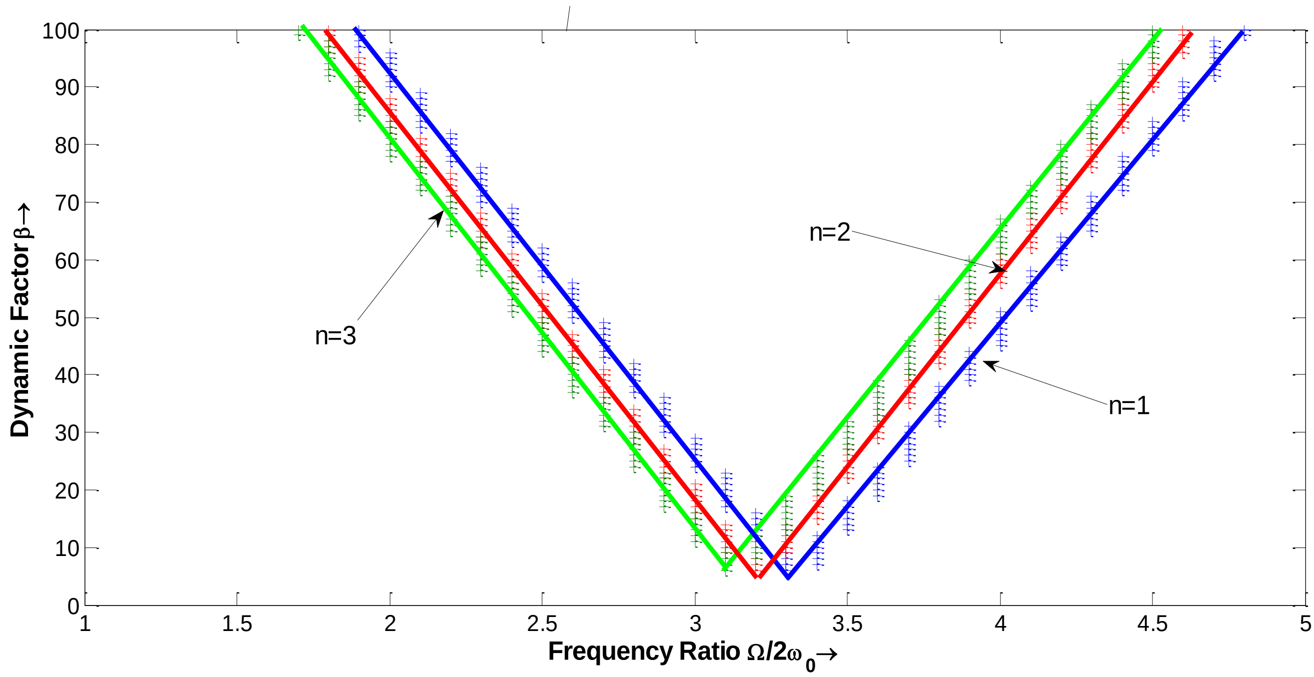


Figure- 5.30 Instability Regions: Core loss factor(η_c)=0.18,Power law index(n)=1, 2, 3 Core thickness parameter(t_2/t_1) = 0.5. First mode of vibration for simple resonance case.

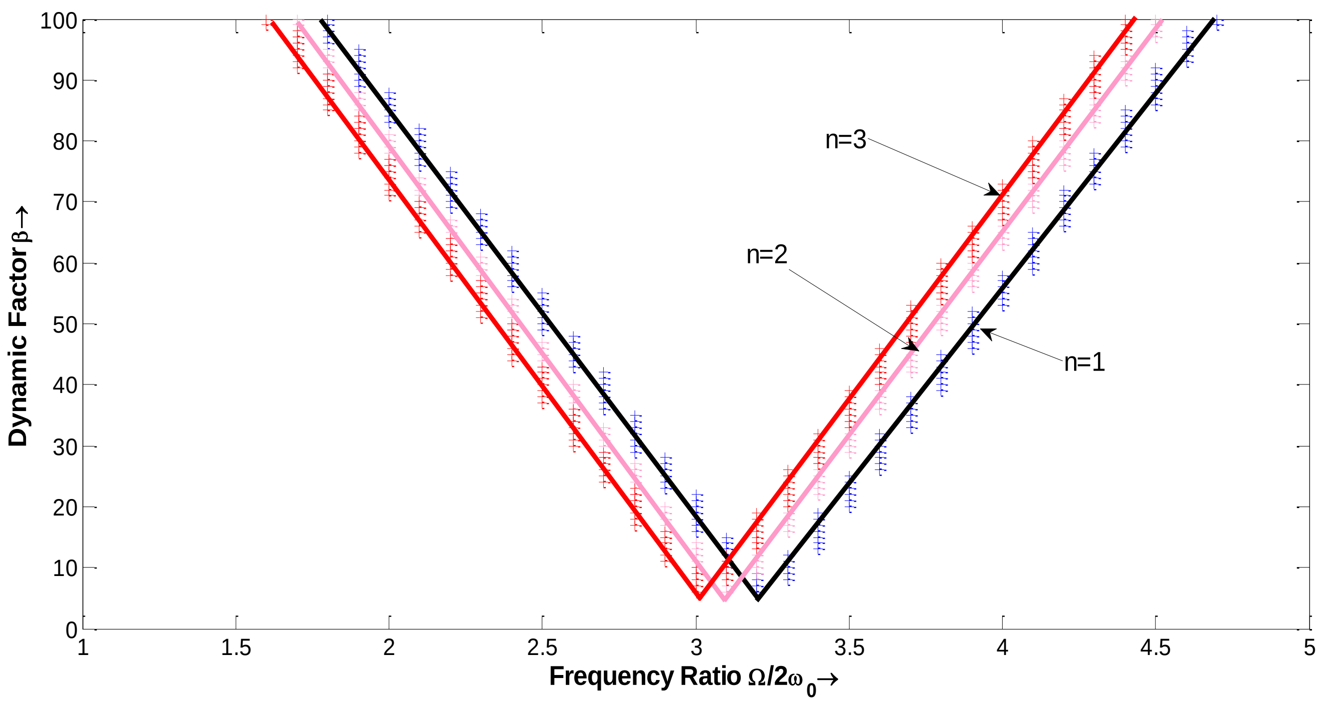


Figure-5.31 Instability Regions: core loss factor(η_c)=0.18, power law index(n)=1, 2, 3 core thickness parameter(t_2/t_1) = 1. First mode of vibration for simple resonance case

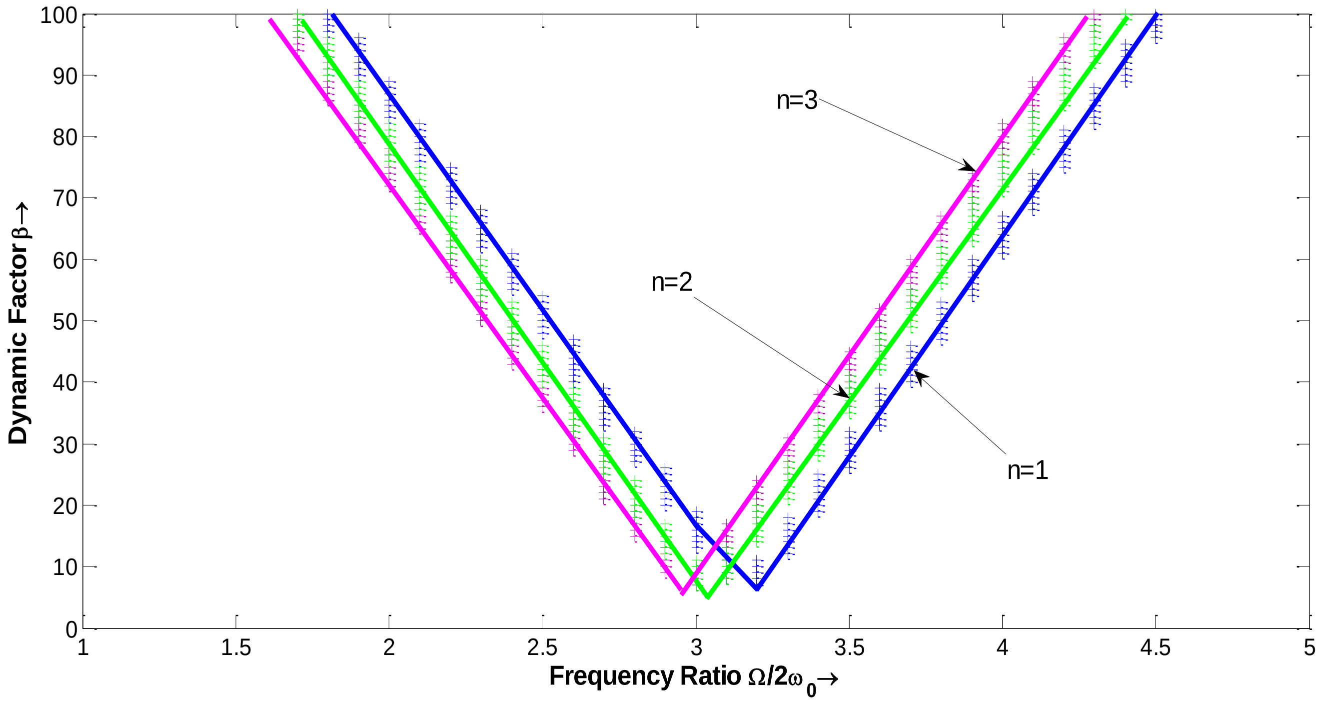


Figure-5.32 Instability Regions: core loss factor(η_c)=0.18, Power law index(n)=1, 2, 3. Core thickness parameter(t_2/t_1) = 2, First mode of vibration for simple resonance case

Figures from 5.30 to 5.32 gives the boundaries of stable and unstable regions of sandwich beam for first mode of vibration with power law index values 1, 2 and 3. Thickness ratio is constant for particular plot and core loss factor is 0.18 constant for all plots.

In all plots, same pattern of variation shifting of boundaries towards lower frequncy of excitation for Power law index values 1, 2 and 3.

Increase in the value of thickness ratio values from 0.5 to 2 is also shifts the instability boundaries towards lower frequency of excitation which exhibits more probability of instability of beam.

CONCLUSIONS

The present study deals with theoretical investigation of Dynamic stability analysis of a sandwich beam with FGM constraining layer under axial pulsating load. Finite element has been used to model the sandwich beam. The stability regions for sandwich beam have been established by using stability criteria proposed by Saito and Otomi.

The following conclusions can be drawn from the present work

1. With increase in power law index, buckling load parameter decreases for all core thickness ratios and core loss factors.

2. Increase in core thickness ratio parameter, the frequency parameter first decreases and then increases for all power law index values.

3. Increase in the value of core loss factor increases the fundamental loss factor of sandwich beam for all power law index values and core thickness ratios.

4. With the increase in power law index, fundamental loss factor first decreases and then increases and then increase for all thickness ratios and core loss factors.

5. Increase in core thickness ratio, first decreases and then increases the fundamental loss factor.

6. With increase in power law index values, shows probability of instability of beam.

7. With increase in thickness ratio also enhances the instability chances of sandwich beam.

SCOPE FOR FUTURE RESEARCH:

The following cases may be studied in extension for the current work

1. Stability of sandwich beam under various operating conditions such as thermal.

2. Stability of sandwich beam under various boundary conditions.

3. Stability of multilayered sandwich beams.

REFERENCES

1. Bhangale, R. K. and Ganeshan, N., Thermoelastic buckling and vibration behavior of a functionally graded sandwich beam with constrained viscoelastic core. Journal of Sound and Vibration, 295, 294-316, 2006.
2. Ungar. E.E., Loss factors of viscoelastically damped beam structures. Journal of the Acoustical Society of America, 34, 1082-1086, 1962.
3. DiTaranto, R.A., Theory of vibratory bending for elastic and viscoelastic layered finite length beams. Journal of Applied Mechanics, Trans of ASME, 87,881-886, 1965.
4. Asnani, N.T. and Nakra, B.C., Vibration analysis of multilayered beams with alternate elastic and viscoelastic layers. Journal of Institution of Engineers India, Mechanical Engineering Division, 50,187-193, 1970.
5. Chatterjee, A. and Baumgarten, J.R., An analysis of viscoelastic damping characteristics of a simply supported sandwich beam. Journal of Engineering for Industry, Trans of ASME, 93, 1239-1244, 1971.
6. Rao, Y.V.K.S., Vibration of dual core sandwich beams, Journal of Sound and Vibration, 32,175-187, 1974.
7. Asnani, N.T. and Nakra, B.C., Forced Vibration damping characteristics of multilayered beams with constrained viscoelastic layers. J. Eng. (Indus), Trans. ASME, Series B. 98, 895 –901, 1976.
8. Rubayi, N.A. and Charoenree, S., Natural frequencies of vibration of cantilever sandwich beam. Computers and structures.6, 345 – 353, 1976.
9. Rao, D.K. and Stühler, W., Frequency and loss factors of tapped symmetric sandwich beams. J. Appl. Mech., Trans. ASME, 99, 511 – 513, 1977.

10. Rao, Y.V.K.S., Vibration of dual core sandwich beams, Journal of Sound and Vibration, 32,175-187, 1974.
11. Rao, D.K., Frequency and loss factors of sandwich beams under various boundary conditions. J. Mech. Eng. Sci., 20, 271 – 282, 1978.
12. Vaswani, J., Asnani, N.T. and Nakra, B.C., Vibration and damping analysis of curved multilayered beams. Transactions of the CSME, 9, 59-63, 1985.
13. Jones. I.W., Salerno.N.L. and Savacchiop. A., An analytical and experimental evaluation of the damping capacity of sandwich beams with viscoelastic cores. Journal of Engineering for Industry, Trans. of ASME, 89, 438-445, 1967.
14. Johnson, C.D., Kienholz, D.A.,Rogers, L.C., Finite element prediction of damping in beams with constrained viscoelastic layers, Shock and vibration bulletin, 51(1),71-81,1981.
15. Johnson, C.D., Kienholz, D.A., Finite element prediction of damping in structures with constrained viscoelastic layers, AIAA Journal, 20(9), 1284-1290, 1982.
16. Lall. A.K., Asnani, N.T. and Nakra, B.C., Damping analysis of partially covered sandwich beams. Journal of sound and vibration, 123,247-255, 1988.
17. Markus, S., Damping mechanism of beams partially covered by constrained viscoelastic layer, ACTA Technica CSAV 2.179-194, 1974.
18. Dewa, H., Okada, Y. and Nagai, B. Damping characteristics of flexural vibration for partially covered beams with constrained viscoelastic layers. JSME International Journal, series iii, 34,210-217, 1991.
19. Kerwin, E.M.Jr., Damping of flexural waves by a constrained viscoelastic layer. Journal of the Acoustical Society of America, 31, 952 -962, 1959.

20. Kapuria, S., Bhattacharyya, M. and Kumar, A. N., Bending and free vibration response of layered functionally graded beams: A theoretical model and its experimental validation. Composite Structures, 82, 390-402, 2008.
21. Imaino, W.and Harrison, J.C., A comment on constrained layer damping structures with low viscoelastic modulus. Journal of sound and vibration, 149, 354-361, 1991.
22. He, S. and Rao, M.D., Prediction of loss factors of curved sandwich beams. Journal of Sound and Vibration, 159, 101-113, 1992.
23. Simsek, M., Non-linear vibration analysis of a functionally graded Timoshenko beam under action of a moving harmonic load. Composite Structures, 92(10), 2532-2546, 2010.
24. Bhimaraddi, A., Sandwich beam theory and the analysis of constrained layer damping. Journal of sound and vibration, 179, 591-602, 1995.
25. Sakiyama, T., Matsuda, H. and Morita, C., Free vibration analysis of continuous sandwich beams with elastic or viscoelastic cores by applying the discrete Green function. Journal of Sound and Vibration, 198,439-445, 1996.
26. Fasana, A., and Marchesiello, S., Rayleigh-Ritz analysis of sandwich beams. Journal of sound and vibration, 241,643-652, 2001.
27. Banerjee, J.R., Free vibration of sandwich beams using the dynamic stiffness method. Computers and Structures, 81, 1915-1922, 2003.
28. Mead, D. J. and Markus, S., The forced vibration of three – layer, damped sandwich beams with arbitrary boundary conditions. Journal of sound and vibration, 10, 163 – 175, 1969.
29. DiTaranto, R.A., Theory of vibratory bending for elastic and viscoelastic layered finite length beams. Journal of Applied Mechanics, Trans of ASME, 87,881-886, 1965.

30. Asnani, N.T. and Nakra, B.C., Vibration analysis of sandwich beams with viscoelastic core. Jr. Aeronautical Society of India, 24,288-294, 1972.
31. Rao, D. K., Forced vibration of a damped sandwich beam subjected to moving forces. Journal of sound and vibration, 54, 215 – 227, 1977.
32. Kapur, A.D., Nakra, B.C. and Chawla, D.R., Shock response of viscoelastically damped beams. Journal of sound and vibration, 55,351-362, 1977.
33. Sharma, S.R. and Rao, D.K., Static deflection and stresses in sandwich beams under various boundary conditions. Journal of Mechanical Engineering Science, IMechE.24, 11-18, 1982.
34. Frosting, Y. and Baruch, M., Bending of sandwich beams with transversely flexible core. AIAA Journal, 28,523-527, 1990.
35. Alshorbagy, A. E, Eltaher, M. A. and Mahmoud, F. F., Free vibration characteristics of a functionally graded beam by finite element method. Applied Mathematical Modelling, 35, 412-425, 2011.
36. Sun, C.T., Sankar, B.V. and Rao, V.S., Damping and vibration control of unidirectional composite laminates using add-on viscoelastic materials. Journal of Sound and Vibration, 139,277-290, 1990.
37. Salet, T.A.M. and Hamelink, S.A., Numerical analysis of sandwich beams. Computers and structures, 41, 1231-1239, 1991.
38. Ha, K.H., Exact analysis of bending and overall buckling of sandwich beam systems. Computers and structures, 45, 31-40, 1992.
39. Qian, C.and Demao, Z., Vibration analysis theory and application to elastic-viscoelastic composite structures. Computers and structures, 37,585-592, 1990.[91]
40. Bauld, N. R. Jr., Dynamic stability of sandwich columns under pulsating axial loads. AIAA J., 5, 1514 – 1516, 1967.

41. Chonan, S. Vibration and stability of sandwich beams with elastic bonding. Journal of sound and vibration, 85, 525 – 537, 1982.

42. Chonan, S., Vibration and stability of a two- layered beam with imperfect bonding. J. Of Acoustical Society of America, 72, 208 – 213, 1982.

43. Kar, R.C. and Hauger, W., Stability of a sandwich beam subjected to a non-conservative force. Computer and structures, 46,955-958, 1993.

44. Ray, K. and Kar, R.C., Parametric instability of a sandwich beam under various boundary conditions. Computers and structures, 55,857-870, 1995.

45. Ray, K. and Kar, R.C., The parametric instability of partially covered sandwich beams. Journal of Sound and Vibration, 197,137-152, 1996.

46. Ray, K. and Kar, R.C., Parametric instability of a dual cored sandwich beam. Computers and structures, 61,665671, 1996.

47. Ray, K. and Kar, R.C., Parametric instability of a symmetric sandwich beam with higher order effects. Computers and structures, 60,817-824, 1996.

48. Lin, C.Y., and Chen, L.W., Dynamic stability of a rotating beam with a constrained-damping layer. Journal of sound and vibration, 267,209-225, 2002.

49. Saito, H. and Otomi, K., Parametric response of viscoelastically supported beams. Journal of Sound and Vibration, 63, 169 – 178 1979.

50. Mead, D. J. and Markus, S., The forced vibration of three – layer, damped sandwich beams with arbitrary boundary conditions. Journal of sound and vibration, 10, 163 – 175, 1969.

51. Aydogdu, M. and Taskin, V., Free vibration analysis of functionally graded beams with simply supported edges. Materials & Design, 28(5), 1651-1656, 2007.

52. Simsek, M., Vibration analyses of a functionally graded beam under a moving mass by using different beam theories. Composite Structures, 92(4), 904-917, 2010.

53. Simsek, M., Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories. Nuclear Engineering and Design, 240, 697-705, 2010.

54. Akhtar, K. and Kadoli, R., Stress analysis of SUS 304-ceramics functionally graded beams using third order shear deformation theory. IE(I) Journal-MC,89, 31-37, 2008.

55. Chakraborty, A., Gopalakrishnan, S. and Reddy, J. N., A new beam finite element for the analysis of functionally graded materials. International Journalof Mechanical Science, 45, 519-539, 2003.

56. Daryl L.Logan., A first course in Finite element method,Thomson,2007.